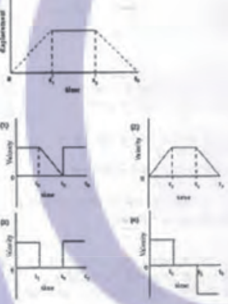
CSIR/UGC/NET MATHEMATICS JUNE 2011

PART A

1. The area of the shaded region in cm^2 is

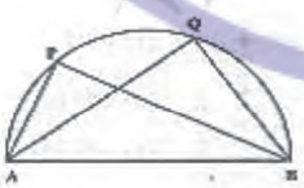
(a) $(\pi - \sqrt{2})$
(b) $(\pi - 2)$
(c) $\left(\frac{\pi}{4} - \frac{\sqrt{2}}{2}\right)$
(d) $(\pi + 2)$

2. Displacement versus time curve for a body is shown in the figure. Select the graph that correctly shows the variation of the velocity with time



3. The angles of a right-angled triangle shaped garden are in arithmetic progression and the smallest side is 10.00 m. The total length of the fencing of the garden in m is
(a) 60.00
(b) 47.32
(c) 12.68
(d) 22.68

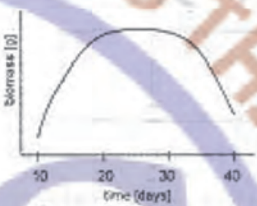
4. AB is the diameter of the semicircle as shown in the diagram. If AQ=2AP, then which of the following is correct?



(a) $\angle APB = \frac{1}{2}\angle AQB$
(b) $\angle APB = \angle AQB$
(c) $\angle APB = \angle AQB$
(d) $\angle APB = \frac{1}{4}\angle AQB$

5. The rabbit population in community A increases at 25% per year while that in B increases at 50% per year. If the present populations of A and B are equal, the ratio of the number of the rabbits in B to that in A after 2 years will be
(a) 1.44
(b) 1.72
(c) 1.90
(d) 1.25

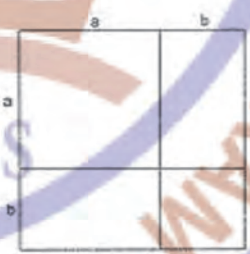
6. Growth of an organism was monitored at regular intervals of time, and is shown in the graph below. Around which time is the rate of growth zero?



(a) Close to day 10
(b) On day 20
(c) Between days 20 and 30
(d) Between days 30 and 40

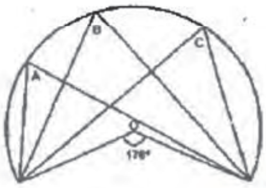
7. A number system consists of digits 0, 1, 2, 3, 4 and 5. What is the decimal equivalent of 15 in this number system?
(a) 15
(b) 23
(c) 11
(d) 12

8. Which of the following is indicated by the accompanying diagram?



(a) $a + ab + ab^2 + \cdots = a/(1-b)$ for $|b| < 1$
(b) $a > b$ implies $a^3 > b^3$
(c) $(a+b)^2 = a^2 + 2ab + b^2$
(d) $a > b$ implies $-a < -b$

9. Three boys A, B and C kicked three balls horizontally from the edge of the roof of a building. The horizontal distances traversed by these balls before hitting the ground are d_A, d_B, d_C respectively, with $d_A > d_B > d_C$. If t_A, t_B and t_C are the times taken to hit the ground respectively, then
(a) $t_A > t_B > t_C$
(b) $t_A < t_B < t_C$
(c) $t_A > t_B < t_C$
(d) $t_A = t_B = t_C$



10. A segment of a circle (slightly greater than a semi-circle, whose center is O) is given below. Identify the correct statement regarding the three angles A, B and C.
   (a) A is equal to B but not equal to C
   (b) A, B and C are equal and have a value 85^o
   (c) A, B and C are unequal
   (d) A, B and C are each equal to 95^o

11. What is the minimum number of cards you need to uncover from the top of a well-shuffled deck of 52 playing cards, to ensure that you have two cards of a suit?
   (a) 41
   (b) 15
   (c) 7
   (d) 5

12. A long cylinder has an axially placed two-bladed fan spinning inside it. Bullets are shot through the cylinder at a constant rate. If the number of blades is increased to four, the number of bullets
   (a) missing the blades is halved
   (b) missing the blades is reduced by one-fourth
   (c) hitting the blades remains the same
   (d) hitting the blades is doubled

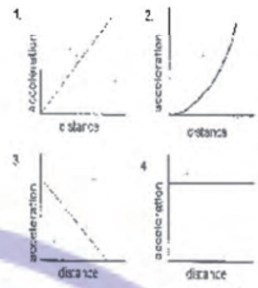
13. The speed of a car increases every minute as shown in the following table. The speed at the end of the 19^{th} minute would be

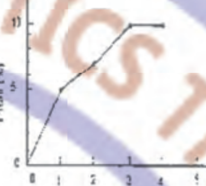
Time (minutes)	Speed (m/sec)
1	1.5
2	3.0
3	3.5
.	.
.	.
24	36.0
25	37.5

   (a) 26.0
   (b) 27.0
   (c) 28.0
   (d) 28.5

14. A ball is dropped from a height h above the surface of the earth. Ignoring air drag, the curve that best represents its variation of acceleration is

15. The cumulative profits of a company since its inception are shown in the diagram. If the net worth of the company at the end of the 4^{th} year is 99 crores, the principal it had started with it is
   (a) 9.9 crores
   (b) 90 crores
   (c) 91 crores
   (d) 9.0 crores





16. Which of the following figures has the longest perimeter?
   (a) A square of radius 10 cm
   (b) A rectangle of sides 12 cm and 9 cm
   (c) A circle of radius 7 cm
   (d) A rhombus of side 9 cm

17. Two pipes fill a tank in 10 hours and 12 hours respectively. While a third pipe empties the tank in 20 hours. If all the three pipes operates simultaneously, in how much time will the tank has filled?
   (a) 7 hours
   (b) 8 hours
   (c) 7 hours 30 minutes
   (d) 8 hours 30 minutes

18. ABCD is a square and F is the midpoint of AB. E is any point on BC such that BE is one-third of BC. If the area of the triangle FBE is 108 sq.cm. Then the approximate length of AC is
   (a) 50 cm
   (b) 51 cm
   (c) 52 cm
   (d) 49 cm

19. Three indistinguishable balls are distributed in three indistinguishable boxes. Then the total number of arrangements will be
   (a) Three
   (b) Six
   (c) Twenty Seven
   (d) One

20. There are 10 participants in a seminar. A list is to be prepared showing the sequence in which the 10 participants will speak. In how many ways can the list be prepared if one particular speaker "P" speak before another particular speaker "Q".
   (a) 5!
   (b) $2 \times 5!$
   (c) $5 \times 9!$
   (d) 9!

PART B

21. Let W be the vector space of all real polynomials of degree at most 3. Define $T : W \to W$ by $(Tp)(x) = p'(x)$ where p' is the derivative of p. The matrix of T in the basis $\{1, x, x^2, x^3\}$, considered as column vectors, is given by

1. $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$ 2. $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{pmatrix}$

3. $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ 4. $\begin{pmatrix} 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

22. Using the fact that

$$\sum_1^\infty \frac{(-1)^{n+1}}{n} = \log 2, \ \sum_1^\infty \frac{(-1)^n}{n(n+1)} \text{ equals}$$

1. $1 - 2\log 2$
2. $1 + \log 2$
3. $(\log 2)^2$
4. $-(\log 2)^2$

23. The reaction time to a stimulus X (in seconds) is distributed normally in

group 1 with mean 2 and variance 8;
group 2 with mean 4 and variance 1.

The two groups appear in equal proportions. x is an observable value of X. The best discriminant function (in the sense of minimizing misclassification probabilities) is to classify into group

1. 2 if $x > 3$; otherwise in group 1
2. 1 if $x > 3$; otherwise in group 2
3. 2 if $0 \le x \le \frac{8}{3}$; otherwise in group 1

24. For $V = (V_1, V_2) \in \mathbb{R}^2$ and $W = (W_1, W_2) \in \mathbb{R}^2$, consider the determinant map

$\det : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ defined by

$\det(V, W) = V_1W_2 - V_2W_1$

Then the derivative of the determinant map at $(V, W) \in \mathbb{R}^2 \times \mathbb{R}^2$ evaluated on $(H, K) \in \mathbb{R}^2 \times \mathbb{R}^2$ is

1. $\det(H, W) + \det(V, K)$
2. $\det(H, K)$
3. $\det(H, V) + \det(W, K)$
4. $\det(V, H) + \det(K, W)$

25. Consider the LP problem
maximize $x_1 + x_2$
subject to

$x_1 - 2x_2 \le 10$
$x_2 - 2x_1 \le 10$
$x_1, x_2 \ge 0$

Then

1. The LP problem admits an optimal solution
2. The LP problem is unbounded
3. The LP problem admits no feasible solution
4. The LP problem admits a unique feasible solution

26. A system of 5 identical units consists of two parts A and B which are connected in series. Part A has 2 units connected in parallel and part B has 3 units connected in parallel. All the 5 units function independently with probability of failure $\frac{1}{2}$. Then the reliability of the system is

1. $\frac{31}{32}$
2. $\frac{11}{32}$
3. $\frac{1}{32}$
4. $\frac{21}{\ldots}$

27. Consider a group G. Let Z(G) be its centre, i.e., $Z(G) = \{g \in G: gh = hg \text{ for all } h \in G\}$. For $n \in \mathbb{N}$, the set of positive integers, define

$J_n = \{(g_1,\cdots,g_n) \in Z(G) \times \ldots \times Z(G) :$

$g_1 \cdots g_n = e\}$.

As a subset of the direct product group $G \times \cdots \times G$ (n times direct product of the group G), J_n is

1. not necessarily a subgroup.
2. a subgroup but not necessarily a normal subgroup.
3. a normal subgroup.
4. isomorphic to the direct product $Z(G) \times \cdots \times Z(G)$ ((n–1) times).

28. The number of elements in the set $\{ m : 1 \le m \le 1000,$ m and 1000 are relatively prime$\}$ is

1. 100
2. 250
3. 300
4. 400

29. Let $X_1, X_2,\cdots,X_n$, $n \ge 2$, be i.i.d. observations from $N(0, \sigma^2)$ distribution, where $0 < \sigma^2 < \infty$ is an unknown parameter. Then the uniformly minimum variance unbiased estimate for σ^2 is

1. $\frac{1}{n}\sum_{i=1}^{n} X_i^2$
2. $\frac{1}{n-1}\sum_{i=1}^{n} X_i^2$
3. $\frac{1}{n}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2$
4. $\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2$

30. The degree of the extension $\mathbb{Q}\left(\sqrt{2} + \sqrt[3]{2}\right)$

Over the field $\mathbb{Q}\left(\sqrt{2}\right)$ is

1. 1
2. 2
3. 3
4. 6

31. Suppose $X_1, X_2, \cdots$ is an i.i.d. sequence of random variables with common variance

$\sigma^2 > 0$. Let $Y_n = \frac{1}{n}\sum_{i=1}^{n} X_{2i-1}$ and

$$Z_n = \frac{1}{n}\sum_{i=1}^{n} X_{2i}$$

Then the asymptotic distribution (as $n \to \infty$) of $\sqrt{n}(Y_n - Z_n)$ is

1. $N(0,1)$
2. $N(0, \sigma^2)$
3. $N(0, 2\sigma^2)$
4. degenerate at 0

32. Consider the function $f(x) = e^{-x}$ and its Taylor approximation g(x) of degree 3. For $x = \frac{1}{3}$, g(x) is

1. positive and less than 1
2. negative and less than –2
3. positive and greater than 1
4. less than 1 but greater than 0.75

33. Let $x=10$ be an observation on the hypergeometric random variable X, namely

$$P(X = x) = \frac{\binom{m}{x}\binom{N-m}{n-x}}{\binom{N}{n}}, x = 0,1,\cdots,$$

$\min\{m,n\}$ and $n - x \le N - m$

where $m=40$, $n=30$ and N is an unknown parameter. The maximum likelihood estimator of N is

1. 75
2. 120
3. 60
4. not unique

34. The number of 4 digit numbers with no two digits common is

1. 4536
2. 3024
3. 5040
4. 4823

35. Consider an aperiodic Markov chain with state space S and with stationary transition probability matrix $P = ((p_{ij}))$, $i,j \in S$. Let the n-step transition probability matrix be denoted by $P^n = ((p_{ij}^{\,n}))$, $i, j \in S$. Then which of the following statements is true?

1. $\lim_{n\to\infty} p_{ii}^n = 0$ only if i is transient.
2. $\lim_{n\to\infty} p_{ii}^n > 0$ if and only if i is recurrent.
3. $\lim_{n\to\infty} p_{ij}^n = \lim_{n\to\infty} p_{jj}^n$ if i and j are in the same communicating class.
4. $\lim_{n\to\infty} p_{ij}^n = \lim_{n\to\infty} p_{ii}^n$ if i and j are in the same communicating class.

36. Suppose that we have i.i.d. observations $(X_1, Y_1), (X_2, Y_2), \cdots, (X_n, Y_n)$, $n \geq 3$, where X_i and Y_i are independent normal random variables. Consider τ = the sample Kendall's rank correlation coefficient computed from this data. Then which of the following is correct?

1. $P(\tau > 0) > \frac{1}{2}$
2. $P(\tau < 0) > \frac{1}{2}$
3. $E(\tau) = 0$
4. $E(\tau) \neq 0$

37. Let $f: \mathbb{C} \to \mathbb{C}$ be a complex valued function given by
$f(z) = u(x,y) + iv(x,y)$.
Suppose that $v(x, y) = 3xy^2$. Then

1. f cannot be holomorphic on $\mathbb{C}$ for any choice of u.
2. f is holomorphic on $\mathbb{C}$ for a suitable choice of u.
3. f is holomorphic on $\mathbb{C}$ for all choices of u.
4. v is not differentiable as a function of x and y.

38. The power series $\sum_{0}^{\infty} 2^{-n} z^{2n}$ converges if

1. $|z| \leq 2$
2. $|z| < 2$
3. $|z| \leq \sqrt{2}$
4. $|z| < \sqrt{2}$

39. Batteries for torch lights are packed in boxes of 10 and a lot contains 10 boxes. A quality inspector randomly chooses a box and then checks two batteries selected randomly without replacement from that box. The lot will be rejected if any one of the two chosen batteries turns out to be defective. Suppose that 9 of the 10 boxes in the lot contain no defective batteries and only one box contains 2 defective ones. What is the probability that the lot will NOT be passed by the Inspector?

1. $\frac{197}{4950}$
2. $\frac{98}{2475}$
3. $\frac{8}{225}$
4. $\frac{17}{450}$

40. Let $X(t)$ be the number of customers in an M/M/1 queueing system with arrival rate 3 and service rate 6. Which of the following is true?

1. $\lim_{t\to\infty} P(X(t) \geq 5) = 0$
2. $\lim_{t\to\infty} P(X(t) \geq 5) = \frac{1}{32}$
3. $\lim_{t\to\infty} P(X(t) \geq 5) = \frac{31}{32}$
4. $\lim_{t\to\infty} P(X(t) \geq 5) = 1$

41. Which of the following is/are correct

1. A free particle in $\mathbb{R}^3$ can have infinite degrees of freedom
2. The number of degree of freedom of N particles is greater than 3N
3. A system of N particles with k constants has 3N + k degrees of freedom
4. A system consisting of three point masses connected by three rigid massless rods has six degrees of freedom.

42. The unit digit of 2^{100} is

1. 2
2. 4
3. 6
4. 8

43. Let G be a group of order 77.Then the center of G is isomorphic to

1. $\mathbb{Z}_{(1)}$
2. $\mathbb{Z}_{(7)}$
3. $\mathbb{Z}_{(11)}$
4. $\mathbb{Z}_{(77)}$

44. Let $f_n(x) = x^{1/n}$ for $x \in [0,1]$. Then

1. $\lim_{n\to\infty} f_n(x)$ exists for all $x \in [0,1]$.
2. $\lim_{n\to\infty} f_n(x)$ defines a continuous function on [0,1].
3. $\{f_n\}$ converges uniformly on [0,1].
4. $\lim_{n\to\infty} f_n(x) = 0$ for all $x \in [0,1]$.

45. To examine whether two different skin creams, A and B, have different effect on the human body n randomly chosen persons were enrolled in a clinical trial. Then cream A was applied to one of the randomly chosen arms of each person, cream B to the other. What kind of a design is this?

1. Completely Randomized Design
2. Balanced Incomplete Block Design
3. Randomized Block Design
4. Latin Square Design

46. Consider the ODE

$$u''(t) + P(t)u'(t) + Q(t)u(t) = R(t),\ t \in [0,1]$$

There exist continuous functions P, Q and R defined on [0, 1] and two solutions u_1 and u_2 of this ODE such that the Wronskian W of u_1 and u_2 is

1. $W(t) = 2t - 1,\quad 0 \le t \le 1$
2. $W(t) = \sin 2\pi t,\quad 0 \le t \le 1$
3. $W(t) = \cos 2\pi t,\quad 0 \le t \le 1$
4. $W(t) = 1,\quad 0 \le t \le 1$

47. Let I_1 be the ideal generated by x^4+3x^2+2 and I_2 be the ideal generated by $x^3 + 1$ in $\mathbb{Q}[x]$. If $F_1 = \mathbb{Q}[x]/I_1$ and $F_2 = \mathbb{Q}[x]/I_2$, then

1. F_1 and F_2 are fields.
2. F_1 is a field, but F_2 is not a field.
3. F_1 is not a field while F_2 is a field.
4. neither F_1 nor F_2 is a field.

48. For the Volterra type linear integral equation

$$\phi(x) = x + 2\int_0^x e^{x-\zeta}\phi(\zeta)\,d\zeta,$$

the resolvent kernel $R(x,\zeta;2)$ of the kernel $e^{x-\zeta}$ is

1. $(x-\zeta)^2 e^{2(x-\zeta)}$,
2. $(x-\zeta)e^{x-\zeta}$
3. $e^{3(x-\zeta)}$
4. $e^{(x-\zeta)}$

49. A general solution of the second order equation $4u_{xx} - u_{yy} = 0$ is of the form $u(x,y) =$

1. $f(x) + g(y)$
2. $f(x + 2y) + g(x - 2y)$
3. $f(x + 4y) + g(x - 4y)$
4. $f(4x + y) + g(4x - y)$

where f and g are twice differentiable functions.

50. The dimension of the vector space of all symmetric matrices of order $n \times n$ $(n \ge 2)$ with real entries and trace equal to zero is

1. $(n^2 - n)/2 - 1$
2. $(n^2 - 2n)/2 - 1$
3. $(n^2 + n)/2 - 1$
4. $(n^2 + 2n)/2 - 1$

51. Let D be a non-zero $n \times n$ real matrix with $n \ge 2$. Which of the following implications is valid?

1. det (D) = 0 implies rank (D) = 0
2. det (D) = 1 implies rank (D) $\neq$ 1
3. rank (D) = 1 implies det (D) $\neq$ 0
4. rank (D) = n implies det (D) $\neq$ 0

52. The number of characteristic curves of the PDE
$(x^2+2y)\,u_{xx}+(y^3-y+x)u_{yy}+x^2\,(y-1)u_{xy}+3u_x+u=0$
passing through the point $x=1, y=1$ is

1. 0
2. 1
3. 2
4. 3

53. Let $I=\{1\}\cup\{2\}\subset\mathbb{R}$. For $x\in\mathbb{R}$, let $\varphi(x)=\text{dist}\,(x, I)=\inf\{|x-y| : y\in I\}$. Then

1. φ is discontinuous somewhere on $\mathbb{R}$.
2. φ is continuous on $\mathbb{R}$ but not differentiable only at $x=1$.
3. φ is continuous on $\mathbb{R}$ but not differentiable only at $x=1$ and 2.
4. φ is continuous on $\mathbb{R}$ but not differentiable only at $x=1$, 3/2 and 2.

54. Let $S=\left\{A : A=\left[a_{ij}\right]_{5\times5},\ a_{ij}=0 \text{ or } 1\ \forall i,j,\ \sum_j a_{ij}=1\ \forall i \text{ and } \sum_i a_{ij}=1\ \forall j\right\}$.
Then the number of elements in S is

1. 5^2
2. 5^5
3. 5!
4. 55

55. Suppose X is a random variable with E(X) = Var(X). Then the distribution of X

1. is necessarily Poisson.
2. is necessarily Exponential.
3. is necessarily Normal.
4. cannot be identified from the given data.

56. Let $A=\{x^2 : 0<x<1\}$ and $B=\{x^3 : 1<x<2\}$. Which of the following statements is true?

1. There is a one to one, onto function from A to B.
2. There is no one to one, onto function from A to B taking rationals to rationals.
3. There is no one to one function from A to B which is onto.
4. There is no onto function from A to B which is one to one.

57. Let P be a polynomial of degree N, with $N\geq 2$. Then the initial value problem $u'(t)=P\,(u(t)),\ u(0)=1$ has always

1. a unique solution in $\mathbb{R}$.
2. N number of distinct solution in $\mathbb{R}$.
3. no solution in any interval containing 0 for some P.
4. a unique solution in an interval containing 0.

58. Let ζ be a primitive fifth root of unity. Define

$$A=\begin{pmatrix}\zeta^{-2} & 0 & 0 & 0 & 0\\ 0 & \zeta^{-1} & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & \zeta & 0\\ 0 & 0 & 0 & 0 & \zeta^2\end{pmatrix}$$

For a vector $\mathbf{v}=(v_1, v_2, v_3, v_4, v_5)\in\mathbb{R}^5$, define $|\mathbf{v}|_A=\sqrt{|\mathbf{v}A\mathbf{v}^T|}$ where $\mathbf{v}^T$ is transpose of $\mathbf{v}$. If $\mathbf{w}=(1,-1,1,1,-1)$, then $|\mathbf{w}|_A$ equals

1. 0
2. 1
3. −1
4. 2

59. The variational problem of extremizing the functional

$$I(y(x))=\int_0^{2\pi}\left[\left(\frac{d}{dx}y\right)^2-y^2\right]dx;\ y(0)=1, y(2\pi)=1$$

has

1. a unique solution
2. exactly two solutions
3. an infinite number of solutions
4. no solution

60. The set $\left\{\frac{1}{n}\sin\frac{1}{n} : n\in\mathbb{N}\right\}$ has

1. one limit point and it is 0
2. one limit point and it is 1
3. one limit point and it is −1
4. three limit points and these are −1, 0 and 1

PART C

Unit I

61. Let M be the vector space of all 3 × 3 real matrices and let

$$A=\begin{pmatrix}2 & 1 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{pmatrix}$$

Which of the following are subspaces of M?

1. $\{X \in M : XA = AX\}$
2. $\{X \in M : X + A = A + X\}$
3. $\{X \in M : \text{trace}(AX) = 0\}$
4. $\{X \in M : \det(AX) = 0\}$

62. Let N be a 3 × 3 nonzero matrix with the property $N^3 = 0$. Which of the following is/are true?

1. N is not similar to a diagonal matrix.
2. N is similar to a diagonal matrix.
3. N has one non-zero eigenvector.
4. N has three linearly independent eigenvector.

63. Let $\{f_n\}$ be a sequence of integrable functions defined on an interval [a, b]. Then

1. If $f_n(x) \to 0$ a.e., then $\int_a^b f_n(x)dx \to 0$
2. If $\int_a^b f_n(x)dx \to 0$, then $f_n(x) \to 0$ a.e.
3. If $f_n(x) \to 0$ a.e. and each f_n is a bounded function, then $\int_a^b f_n(x)dx \to 0$
4. If $f_n(x) \to 0$ a.e. and the f_n's are uniformly bounded, then $\int_a^b f_n(x)dx \to 0$

64. Let W = {p(B) : p is a polynomial with real coefficients}, where $B=\begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 0 & 0\end{pmatrix}$

The dimension d of the vector space W satisfies

1. $4 \le d \le 6$
2. $6 \le d \le 9$
3. $3 \le d \le 8$
4. $3 \le d \le 4$

65. Let X denote the two-point set {0, 1} and write $X_j = \{0, 1\}$ for every j = 1, 2, 3,... Let $Y=\prod_{j=1}^{\infty} X_j$. Which of the following is/are true?

1. Y is a countable set.
2. Card Y = card [0,1].
3. $\bigcup_{n=1}^{\infty}\left(\prod_{j=1}^{n} X_j\right)$ is uncountable.
4. Y is uncountable.

66. Which of the following is/are metrics on $\mathbb{R}$?

1. $d(x, y) = \min(x, y)$
2. $d(x, y) = |x - y|$
3. $d(x, y) = |x^2 - y^2|$
4. $d(x, y) = |x^3 - y^3|$

67. For $x = (x_1, x_2, \ldots, x_d) \in \mathbb{R}^d$, and $p \ge 1$, define

$$\|x\|_p = \left(\sum_{j=1} |x_j|^p\right)^{1/p}$$ and

$\|x\|_\infty = \max\{|x_j| : j = 1, 2, \ldots d\}$. Which of the following inequalities hold for all $x \in \mathbb{R}^d$?

1. $\|x\|_1 \ge \|x\|_2 \ge \|x\|_\infty$
2. $\|x\|_1 \le d\,\|x\|_\infty$
3. $\|x\|_1 \le \sqrt{d}\,\|x\|_\infty$
4. $\|x\|_1 \le \sqrt{d}\,\|x\|_2$

68. Which of the following is/are correct?

1. $n\log\left(1+\frac{1}{n+1}\right)\to 1$ as $n\to\infty$
2. $(n+1)\log\left(1+\frac{1}{n}\right)\to 1$ as $n\to\infty$
3. $n^2\log\left(1+\frac{1}{n}\right)\to 1$ as $n\to\infty$
4. $n\log\left(1+\frac{1}{n^2}\right)\to 1$ as $n\to\infty$

69. Let T be a linear transformation on the real vector space $\mathbb{R}^n$ over $\mathbb{R}$ such that $T^2 = \lambda T$ for some $\lambda\in\mathbb{R}$. Then

1. $\|Tx\| = |\lambda|\,\|x\|$ for all $x\in\mathbb{R}^n$. ✓
2. If $\|Tx\| = \|x\|$ for some non-zero vector $x\in\mathbb{R}^n$, then $\lambda = \pm 1$
3. $T = \lambda I$ where I is the identity transformation on $\mathbb{R}^n$.
4. If $\|Tx\| > \|x\|$ for a nonzero vector $x\in\mathbb{R}^n$, then T is necessarily singular.

70. Let $a_{ij} = a_i a_j$, $1 \le i, j \le b$, where $a_1, \ldots, a_n$ are real numbers. Let $A = ((a_{ij}))$ be the $n \times n$ matrix $((a_{ij}))$. Then

1. It is possible to choose $a_1,\ldots,a_n$ so as to make the matrix A non-singular.
2. The matrix A is positive definite if $(a_1,\ldots,a_n)$ is a nonzero vector
3. The matrix A is positive semidefinite for all $(a_1,\ldots,a_n)$.
4. For all $(a_1,\ldots,a_n)$, zero is an eigenvalue of A.

71. Consider the map $f: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(x,y) = (3x - 2y + x^2, 4x + 5y + y^2)$ Then

1. f is discontinuous at (0, 0).
2. f is continuous at (0, 0) and all directional derivatives exist at (0, 0).
3. f is differentiable at (0, 0) but the derivative $Df(0,0)$ is **not** invertible.
4. f is differentiable at (0, 0) and the derivative $Df(0, 0)$ **is** invertible

72. Let $x, y \in \mathbb{C}^n$. Consider

$$f(x,y)=\sup_{\theta,\varphi}\left\|e^{i\theta}x-e^{i\varphi}y\right\|_2,\ \theta,\phi\in\mathbb{R}.$$

Which of the following is/are correct?

1. $f(x,y)\le\|x\|^2+\|y\|^2-2Re\left|\langle x,y\rangle\right|$
2. $f(x,y)\le\|x\|^2+\|y\|^2+2Re\left|\langle x,y\rangle\right|$
3. $f(x,y)=\|x\|^2+\|y\|^2+2\left|\langle x,y\rangle\right|$
4. $f(x,y)\ge\|x\|^2+\|y\|^2-2Re\langle x,y\rangle$

73. Which of the following subsets of $\mathbb{R}^2$ are convex?

1. $\{(x, y) : |x| \le 5\ |y| \le 10\}$
2. $\{(x, y) : x^2 + y^2 = 1\}$
3. $\{(x, y) : y \ge x^2\}$
4. $\{(x, y) : y \le x^2\}$

74. Consider the function

$f(x) = |\cos x|+|\sin(2-x)|$.

At which of the following points is f <u>not</u> differentiable?

1. $\left\{(2n+1)\frac{\pi}{2} : n\in\mathbb{Z}\right\}$
2. $\{n\pi : n\in\mathbb{Z}\}$
3. $\{n\pi+2 : n\in\mathbb{Z}\}$
4. $\left\{\frac{n\pi}{2} : n\in\mathbb{Z}\right\}$

75. Let

$F = \{f:\mathbb{R}\to\mathbb{R} : |f(x)-f(y)| \le K|(x-y)|^{\alpha}\}$

for all $x, y\in\mathbb{R}$ and for some $\alpha > 0$ and some $K > 0$.

Which of the following is/are true?

1. every $f\in F$ is continuous
2. every $f\in F$ is uniformly continuous
3. every differentiable function f is in F
4. every $f\in F$ is differentiable

76. Suppose A, B are $n \times n$ positive definite matrices and I be the $n \times n$ identity matrix. Then which of the following are positive definite.

1. A + B
2. ABA^*
3. A^2+I
4. AB

77. If $\{x_n\}$ and $\{y_n\}$ are sequences of real numbers, which of the following is/are true?

1. $\limsup_n (x_n+y_n) \le \limsup_n x_n + \limsup_n y_n$
2. $\limsup_n (x_n+y_n) \ge \limsup_n x_n + \limsup_n y_n$
3. $\liminf_n (x_n+y_n) \le \liminf_n x_n + \liminf_n y_n$
4. $\liminf_n (x_n+y_n) \ge \liminf_n x_n + \liminf_n y_n$

78. Which of the following sets are dense in $\mathbb{R}$ with respect to the usual topology.

1. $\{(x, y) \in \mathbb{R}^2 : x \in \mathbb{N}\}$
2. $\{(x, y) \in \mathbb{R}^2 : x + y \text{ is a rational number}\}$
3. $\{(x, y) \in \mathbb{R}^2 : x + y^2 = 5\}$
4. $\{(x, y) \in \mathbb{R}^2 : xy \neq 0\}$

Unit II

79. Let $H = \{e, (1,2)(3,4)\}$ and $K = \{e, (1,2)(3,4), (1,3)(2,4), (1,4)(2,3)\}$ be subgroups of S_4, where e denotes the identify element of S_4. Then

1. H and K are normal subgroups of S_4
2. H is normal in K and K is normal in A_4
3. H is normal in A_4 but not normal in S_4
4. K is normal in S_4 but H is not

80. Let $f : \mathbb{D} \to \mathbb{D}$ be holomorphic with $f(0) = ½$ and $f(1/2) = 0$, where $\mathbb{D} = \{z : |z| \le 1\}$. Which of the following is correct?

1. $|f'(0)| \le 3/4$
2. $|f'(1/2)| \le 4/3$
3. $|f'(0)| \le 3/4$ and $|f'(1/2)| \le 4/3$
4. $f(z) = z, z \in \mathbb{D}$

81. Let <p(x)> denote the ideal generated by the polynomial p(x) in $\mathbb{Q}[x]$. If $f(x) = x^3 + x^2 + x + 1$ and $g(x) = x^3 - x^2 + x - 1$, then

1. $\langle f(x)\rangle + \langle g(x)\rangle = \langle x^3 + x\rangle$
2. $\langle f(x)\rangle + \langle g(x)\rangle = \langle f(x)\cdot g(x)\rangle$
3. $\langle f(x)\rangle + \langle g(x)\rangle = \langle x^2 + 1\rangle$
4. $\langle f(x)\rangle + \langle g(x)\rangle = \langle x^4 - 1\rangle$

82. At $z = 0$ the function $f(z) = \frac{e^z + 1}{e^z - 1}$ has

1. a removable singularity.
2. a pole.
3. an essential singularity.
4. the residue of $f(z)$ at $z = 0$ is 2.

83. Define
$$H^+ = \{z \in \mathbb{C} : y > 0\}$$
$$H^- = \{z \in \mathbb{C} : y < 0\}$$
$$L^+ = \{z \in \mathbb{C} : x > 0\}$$
$$L^- = \{z \in \mathbb{C} : x < 0\}$$

The function $f(z) = \frac{z}{3z+1}$

1. maps H^+ onto H^+ and H^- onto H^-
2. maps H^+ onto H^- and H^- onto H^+
3. maps H^+ onto L^+ and H^- onto L^-
4. maps H^+ onto L^- and H^- onto L^+

84. Consider three subsets of $\mathbb{R}^2$, namely
$$A_1 = \{(x, y) : x^2 + y^2 \le 1\}$$
$$A_2 = \{(1, y) : y \in \mathbb{R}\}$$
$$A_3 = \{(0, 2)\}$$

Then there always exists a continuous real-valued function f on $\mathbb{R}^2$ such that

$$f(x) = a_j \text{ for } x \in A_j, j = 1, 2, 3$$

1. if and only if at least two of the numbers a_1, a_2, a_3 are equal
2. if $a_1 = a_2 = a_3$
3. for all real values of a_1, a_2, a_3
4. if and only if $a_1 = a_2$

85. Let $G = \mathbb{Z}_{10} \times \mathbb{Z}_{15}$. Then

1. G contains exactly one element of order 2
2. G contains exactly 5 elements of order 3
3. G contains exactly 24 elements of order 5
4. G contains exactly 24 elements of order 10

86. Let I_1 be the ideal generated by $x^2 + 1$ and I_2 be the ideal generated by $x^3 - x^2 + x - 1$ in $\mathbb{Q}[x]$.

If $R_1 = \mathbb{Q}[x]/I_1$ and $R_2 = \mathbb{Q}[x]/I_2$,then

1. R_1 and R_2 are fields.
2. R_1 is a field and R_2 is not a field.
3. R_1 is an integral domain, but R_2 is not an integral domain.
4. R_1 and R_2 are not integral domains.

87. Consider the set

$X = (-\infty, 0] \cup \left\{\frac{1}{n} : n \in \mathbb{N}\right\} \subseteq \mathbb{R}$ with the subspace topology. Then

1. 0 is an isolated point.
2. $(-2, 0]$ is an open set.
3. 0 is a limit point of the subset $\left\{\frac{1}{n} :\ n \in \mathbb{N}\right\}$
4. $(-2, 0)$ is an open set.

88. The space C [0, 1] of continuous functions on [0, 1] is complete with respect to which of the following

1. $\|f\|_\infty = \sup\{|f(x)| : x \in [0, 1]\}$
2. $\|f\|_2 = \left(\int_0^1 |f(x)|^2\, dx\right)^{1/2}$
3. $\|f\|_{\infty,\,1/2} = \|f\|_\infty + |f(1/2)|$
4. $\|f\|_\infty$ and $\|f\|_{\infty,\,1/2}$

89. Let f be an entire function. If $Re\, f$ is bounded then

1. $Im\, f$ is constant
2. f is constant
3. $f \equiv 0$
4. f' is a non zero constant

90. Let $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ be the unit disc. Let $f : \mathbb{D} \to \mathbb{C}$ be an analytic function satisfying $f\left(\frac{1}{n}\right) = \frac{2n}{3n+1}$ for $n \geq 1$. Then

1. $f(0) = 2/3$
2. f has a simple pole at $z = -3$
3. $f(3) = 1/3$
4. no such f exists

Unit III

91. The integral equation, involving a parameter λ,

$$\phi(x) = \cos zx + \lambda \int_0^\pi \cos(x + \zeta)\, d\zeta$$

has

1. a unique solution if $\lambda = 1$, and an infinite number of solution if $\lambda = \frac{2}{\pi}$
2. a unique solution if $\lambda = -1$, and an infinite number of solution if $\lambda = -\frac{2}{\pi}$
3. a unique solution if $\lambda \neq \frac{2}{\pi}$
4. no solution if $\lambda = \pm\frac{2}{\pi}$

92. The PDE

$$\left.\begin{aligned} &u_{xx} + u_{yy} + \lambda u = 0, \quad 0 < x, y < 1 \\ &u(x,0) = u(x,1) = 0, \quad 0 \leq x \leq 1 \\ &u(0,y) = u(1,y) = 0, \quad 0 \leq y \leq 1 \end{aligned}\right\}$$

has

1. a unique solution u for any $\lambda \in \mathbb{R}$.
2. infinitely many solutions for some $\lambda \in \mathbb{R}$.
3. a solution for countably many values of λ.
4. infinitely many solutions for all $\lambda \in \mathbb{R}$.

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93. Consider the force free motion of a rigid body about a fixed point 0. Suppose 3A, 5A and 6A are the principal moments of inertia at 0, and initially the angular velocity has components $\omega_1 = \sqrt{5}$, $\omega_2 = 0$, $\omega_3 = \sqrt{5}$ about the corresponding principal axes; if the body ultimately rotates about the mean axis, then

1. $\omega_1^2 + \omega_2^2 = 5$
2. $5\omega_2^2 + g\omega_1^2 = 45$
3. $\omega_3^2 = \omega_1^2$
4. $\omega_2^2 \neq \omega_1^2$

94. Using Euler's dynamical equation for force-free motion of a rigid body, symmetrical about the Z-principal axis, with angular velocity $\overline{\omega} = (\omega_1, \omega_2, \omega_3)$, where ω_i, $i = 1, 2, 3$, are the components along the three principal axes, it follows that

1. ω_i = constant
2. ω_2 = a $\sin(\lambda t + b)$ with a, λ, and b as constant
3. ω_3 = constant
4. $\omega_1^2 + \omega_2^2$ constant

95. Let $y_1(x)$ and $y_2(x)$ form a fundamental set of solutions to the differential equation

$$\frac{d^2y}{dx^2} + p(x)\frac{dy}{dx} + q(x)\,y = 0, \quad a \le x \le b,$$

where p(x) and q(x) are continuous in [a, b], and x_0 is a point in (a, b). Then

1. both $y_1(x)$ and $y_2(x)$ cannot have a local maximum at x_0.
2. both $y_1(x)$ and $y_2(x)$ cannot have a local minimum at x_0.
3. $y_1(x)$ cannot have a local maximum at x_0 and $y_2(x)$ cannot have local minimum at x_0 simultaneously.
4. both $y_1(x)$ and $y_2(x)$ cannot vanish at x_0 simultaneously.

96. Consider the iteration function for Newton's method

$$g(x) = x - \frac{f(x)}{f'(x)}$$

and its application to find (approximate) square root of 2, starting with $x_0 = 2$. Consider the first and the second iterates x_1 and x_2, respectively; then

2 —

1. $1.5 < x_1 \le 2$
2. $1.5 \le x_1 < 2$
3. $x_1 \le 1.5$; $x_2 \le 1.5$
4. $x_1 = 1.5$; $x_2 < 1$

97. Consider a linear system $Ax = b$ with a computed solution x_C; the error and the residue are defined, respectively by

$e = x - x_c$
$r = Ax - Ax_c$ Then

1. A small error necessarily implies a small residue.
2. The error can be large with relatively small residue.
3. The error can be small with relatively large residue.
4. The error and the residue are always equal.

98. A general solution of the PDE $uu_x + yu_y = x$ is of the form

1. $f\left(u^2 - x^2, \dfrac{y}{x+u}\right) = 0$, where $f: \mathbb{R}^2 \to \mathbb{R}$ is C^1 and $\nabla f \neq (0,0)$ at every point
2. $u^2 = g\left(\dfrac{y}{x+u}\right) + x^2$, $g \in C^1(\mathbb{R})$
3. $f(u^2 + x^2) = 0$, $f \in C^1(\mathbb{R})$
4. $f(x + y) = 0$, $f \in C^1(\mathbb{R})$

99. For the boundary value problem,
$y'' + \lambda y = 0$, $y(-\pi) = y(\pi)$,
$y'(-\pi) = y'(\pi)$,
to each eigenvalue λ, there corresponds

1. only one eigenfunction
2. two eigenfunctions
3. two linearly independent eigenfunctions
4. two orthogonal eigenfunctions

100. The Cauchy problem

$$\left.\begin{array}{ll} u_x(x,y)+u_y(x,y)=0 & \text{for } (x,y)\in\mathbb{R}^2 \\ u(x,x)= & \text{for all } x\in\mathbb{R} \end{array}\right\}$$

has

1. a unique solution.
2. a family of straight lines as characteristics.
3. solution which vanishes at (2, 1).
4. infinitely many solutions.

101. The Green's function G(x, ζ), 0 ≤ x, ζ ≤ 1 of the boundary value problem

$y'' + \lambda y = 0, \quad y(0) = 0 = y(1)$ is

1. symmetric in x and ζ
2. continuous at x = ζ
3. $\left.\dfrac{\partial G(x,\zeta)}{\partial x}\right|_{x=\zeta^-} - \left.\dfrac{\partial G(x,\zeta)}{\partial x}\right|_{x=\zeta^+} = -1$
4. $\left.\dfrac{\partial G(x,\zeta)}{\partial x}\right|_{x=\zeta^-} - \left.\dfrac{\partial G(x,\zeta)}{\partial x}\right|_{x=\zeta^+} = 1$

102. In the Ritz method, seeking an extremum of the functional

$$I(y)=\int_{x_0}^{x_1} F\left(x,y,\frac{dy}{dx}\right)dx\ ;\ y(x_0)=a, y(x_1)=b,$$

The coordinate function/or the admissible function $\phi_i(x)$, $i = 1, 2, \ldots$ defined on $[x_0, x_1]$ must be

1. linearly independent
2. continuous
3. smooth
4. linearly independent, smooth and the functional be considered not along admissible curves y = y(x) but only along all possible linear combinations of admissible functions

Unit IV

103. Let X be a random variable taking values in a set E. Let

P(X > a + b | X > a) = P(X > b) for all a, b, ∈ E.

Then which of the following is a possible distribution of X?

1. Poisson
2. Geometric
3. Log-normal
4. Exponential

104. Let $\mathbb{D}$ be a balanced incomplete block design with usual parameters v, b, r, k, λ. Which of the following statements is true?

1. $\mathbb{D}$ is connected if $k \geq 2$.
2. The variance of the best linear unbiased estimator of an elementary treatment contrast under $\mathbb{D}$ is proportional to $2/r$
3. The covariance between the best linear unbiased estimators of a pair of orthogonal treatment contrasts under $\mathbb{D}$ is zero.
4. The efficiency factor of $\mathbb{D}$ relative to a randomized (complete) block design with replication r is strictly smaller than unity.

105. Suppose X and Y are independent N (0, 1) random variables.

Let $U=\dfrac{X}{Y}$ and $V=\dfrac{X}{|Y|}$. Then

1. U and V are independent
2. U and V have the same distribution
3. P(U = V) = 1/2
4. P (U < V) = 1/2

106. Suppose $X \sim N(0, \sigma^2)$, Y has the exponential distribution with mean $2\sigma^2$ and, X and Y are independent. We want to test at level α

$H_0: \sigma^2 \leq 1$ versus $H_1: \sigma^2 > 1$

Then

1. UMP test does not exist
2. UMP test rejects H_0 when $X^2 + Y$ is large
3. UMP test is a chi-square test
4. UMP test is a t-test

107. Suppose $X_1 \sim U(0, \theta)$, $X_2 \sim U(0, 1+\theta)$ and X_1 and X_2 are independent. Then

1. min $\{X_1, X_2\}$ is sufficient for θ
2. max $\{X_1, X_2\}$ is sufficient for θ
3. max $\{X_1, X_2-1\}$ is sufficient for θ
4. max $\{X_1+1, X_2\}$ is sufficient for θ

108. Suppose that we have a data set consisting of 25 observations, where each value is either 5 or 10.

1. The mean of the data cannot be larger than the median.
2. The mean of the data cannot be smaller than the median.
3. The mean and the median for the data will be the same only if the variance of the data is zero.
4. The mean and the median for the data will be different only if the range is 5.

109. Random variables X_1, X_2, X_3 are such that correlation (X_1, X_2) = correlation (X_2, X_3) = correlation $(X_3, X_1) = \rho$.

1. ρ cannot be negative
2. ρ can take any value between -1 and $+1$
3. $\rho \geq -0.5$
4. ρ is either $+1$ or -1

110. Let $\{X_n\}$ be a stationary Markov chain such that
$P(X_{i+1}=1 \mid X_i=1) = p_1 = 1 - P(X_{i+1}=0 \mid X_i=1)$,.

$P(X_{i+1}=1 \mid X_i=0) = p_o = 1 - P(X_{i-1}=0 \mid X_i=0)$,
and $P(X_1=1) = \pi_1 = 1 - P(X_1=0)$
Then

1. $\pi_1 = p_1$
2. $\pi_1 = p_0$
3. $\pi_1 = \dfrac{p_0}{1-p_1+p_0}$
4. $\pi_1 = \dfrac{1}{2}$

111. Suppose $X_1, X_2, \ldots$ is a sequence of i.i.d. random variables where
$P(X_i = 1) = p = 1 - P(X_i = 0)$, $i = 1, 2\ldots$

Let $Z = \dfrac{1}{500}\sum_{i=1}^{500} X_i$ and $\alpha = P(|Z-p| > 0.1)$

Then for all p

1. $\alpha \leq .1$
2. $\alpha \leq .05$
3. $\alpha > .01$
4. $\alpha = 0$

112. Suppose that the LP problem
maximise $c^T x$
subject to
$Ax \leq b$
$x \geq 0$
admits a feasible solution and the dual
minimise $b^T y$
subject to $A^T y \geq c$
$y \geq 0$

admits a feasible solution y_0. Then

1. the dual admits an optimal solution.
2. any feasible solution x_0 of the primal and y_0 of the dual satisfies $b^T y_0 \leq c^T x_0$.
3. the dual problem is unbounded.
4. the primal problem admits an optimal solution.

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113. Which of the following is/are cumulative distribution function(s) (c.d.f.) of random variable(s)?

1. $F_1(x)=\begin{cases}0, & x\le 0\\ e^{-x}, & x>0\end{cases}$
2. $F_2(x)=\begin{cases}0, & x\le 0\\ 1-e^{-x}, & x>0\end{cases}$
3. $F_3(x)=\begin{cases}0, & x\le 0\\ 1, & x>0\end{cases}$
4. $F_4(x)=\begin{cases}0, & x<0\\ 1/2, & 0\le x<1\\ 1, & x\ge 0\end{cases}$

114. Consider a linear model with four observations X_1, X_2, X_3, X_4 such that

$E(X_1) = A+B+C$; $E(X_2) = A$; $E(X_3) = B$; $E(X_4) = A-B-C$

[where A, B, C, D are parameters]. Then

1. B + C is not estimable
2. A, B, C are all estimable
3. A+B+C is estimable
4. X_2 is the Best Linear unbiased estimate of A

115. Let X(t) be the number of customers in an M/M/1 queuing system with arrival rate $\lambda > 0$ and service rate $\mu > 0$.

It is known that $\lim_{t\to\infty} P(X(t)=1)=\frac{1}{4}$.

Which of the following is true?

1. $\lim_{t\to\infty} E(X(t)=1)=\frac{1}{3}$
2. $\lim_{t\to\infty} E(X(t)=1)=\frac{\lambda}{\mu}$
3. $\lim_{t\to\infty} Var(X(t)=1)=\frac{1}{9}$
4. $\lim_{t\to\infty} Var(X(t)=1)=\left(\frac{\lambda}{\mu}\right)^2$

116. Let $X_1, X_2,\ldots$ be i.i.d. observations from $N(\mu,\sigma^2)$ distribution with $-\infty<\mu<+\infty$ and $0<\sigma^2<\infty$ as unknown parameters. Then

1. sample mean is an unbiased estimate for μ but sample median is not an unbiased estimate for μ.
2. both sample mean and sample median are unbiased estimates for μ.
3. sample mean has smaller variance than sample median.
4. sample mean has smaller mean squared error than sample median.

117. Suppose that we have $n \ge 1$ i.i.d. observations $X_1, X_2,\ldots, X_n$ each with a common $N(\mu,1)$ distribution where $\mu \ge 0$ is unknown parameter. Then

1. the maximum likelihood estimate and the uniformly minimum variance unbiased estimate for μ are the same.
2. the minimum variance unbiased estimate for μ is a consistent estimate.
3. for any unbiased estimate for μ, there is another estimate for μ with a smaller mean squared error
4. the maximum likelihood estimate for μ has smaller mean squared error than the estimate obtained by the method of moments.

118. In a survey of a population of $N = nk$ units, a sample of n units is to be drawn by systematic sampling with a random start between 1 and k and selecting every k^{th} unit. Then

1. the sample mean is an unbiased estimate of the population mean.
2. the variance of the sample mean cannot be estimated under this design.
3. if the N population units have been arranged at random, then the sample is equivalent to a simple random sample with replacement.
4. if the N population units have been arranged at random, then the sample is equivalent to a simple random sample without replacement.

119. Suppose that the probability distribution of a discrete random variable X under two possible parameter values is as follows.

Parameter	1	2	3	4
θ_1	.01	.04	.05	.90
θ_2	.80	.10	.05	.05

Test $H_0: \theta = \theta_1$ versus $H_1: \theta = \theta_2$ at level $\alpha=0.05$. Then the most powerful test

1. rejects H_0 if x = 1 or x = 2
2. rejects H_0 if x = 3
3. has power larger than 0.85
4. has power .05

120. In a Bayesian estimation problem of the Poisson mean λ, a gamma prior (with density proportional to $e^{-\beta\lambda}\lambda^{\alpha-1}$) is formulated. There is a sample of size n from the Poisson and the sample mean is $\bar{x}$. The posterior distribution of λ is

1. a gamma distribution
2. a Poisson distribution
3. has mean = $\dfrac{n\bar{x}+\alpha}{n+\beta}$
4. has mean = $(n\bar{x}+\alpha)(n+\beta)$

Answer Key JUNE 2017 JRF NET EXAM

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Roll Number	Booklet Code	Medium	Subject Code	Centre Code
	B	E	4	

अन्वीक्षक के लिए निर्देश
INSTRUCTION FOR INVIGILATOR

कृपया हस्ताक्षर करने से पहले रोल नंबर, प्रश्न पुस्तिका कोड, विषय कोड, केन्द्र कोड व परीक्षार्थी के हस्ताक्षर की जांच कर लें।

Check Roll Number, Booklet Code, Subject Code, Centre Code & Signature of Candidate before signing.

अन्वीक्षक के हस्ताक्षर / Signature of Invigilator

USE DARK HB PENCIL ONLY

Note: Roll Number, Subject Code & Centre Code should be filled in the boxes using the pattern given as:

ANSWER COLUMNS

PART 'A'

प्र.क्र. Q.No.	उत्तर / ANSWER
1	3
2	2
3	3
4	2
5	4
6	2
7	4
8	3
9	3
10	2
11	3
12	3
13	1
14	1
15	3
16	4
17	2
18	2
19	4
20	4

PART 'B'

प्र.क्र. Q.No.	उत्तर / ANSWER
21	2
22	1
23	4
24	1
25	2
26	4
27	3
28	4
29	1
30	3
31	3
32	1
33	2
34	1
35	3
36	3
37	1
38	4
39	4
40	2
41	4
42	3
43	4
44	1
45	2
46	4
47	4
48	3
49	2
50	3
51	2
52	1
53	4
54	3
55	4
56	1
57	4
58	1
* 59	
60	1

PART 'C'

प्र.क्र. Q.No.	उत्तर / ANSWER
61	1, 2, 3
62	1, 3
63	4
64	3, 4
65	2, 4
66	2, 4
67	1, 2, 4
68	1, 2
69	2
70	3, 4
71	2, 4
72	2, 3, 4
73	1, 3
74	1, 3
75	1, 2
76	1, 2, 3
77	1, 4
* 78	
79	2, 4
80	1, 2, 3
81	3
82	2, 4
83	1
84	2, 4
85	1, 3, 4
86	2, 3
87	3, 4
88	1, 3, 4
89	1, 2
90	1, 2, 3
* 91	
92	2, 3
93	2, 3
94	2, 3, 4
95	1, 2, 3, 4
96	2, 3
97	2, 3
98	1, 3
99	2, 3, 4
* 100	
101	1, 2, 4
102	1, 2, 3, 4
103	2, 4
104	1, 3, 4
105	2, 3
106	2, 3
107	3, 4
108	3, 4
109	3
110	3
111	1, 2
112	1, 4
113	2, 4
114	2, 3
115	1, 3
116	2, 3, 4
117	2, 3, 4
118	1, 2, 4
119	1, 3
120	1, 3

* Questions contained error, benefit of Marks given to Candidates who attempted the question