

CSIR/UGC/NET MATHEMATICS MODEL 2009

**PART 'A'**

Three fishermen caught fishes and went to sleep. One of them woke up, took away one fish and  $\frac{1}{3}^{\text{rd}}$  of the remainder as his share, without others' knowledge. Later, the three of them divided the remainder equally. How many fishes were caught?

1. 58
- ✓ 2. 19
3. 76
4. 88

What is the arithmetic mean of  $\frac{1}{1 \times 2}, \frac{1}{2 \times 3}, \frac{1}{3 \times 4}, \frac{1}{4 \times 5}, \dots, \frac{1}{100 \times 101}$ ?

1. 0.01
- ✓ 2.  $\frac{1}{101}$
3. 0.00111...
4.  $\frac{1}{49 \times 50} + \frac{1}{50 \times 51}$

Every time a ball falls to ground, it bounces back to half the height it fell from. A ball is dropped from a height of 1024 cm. The maximum height from the ground to which it can rise after the tenth bounce is

1. 102.4 cm
2. 1.24 cm
- ✓ 3. 1 cm
4. 2 cm

4. A circle of radius 7 units lying in the fourth quadrant touches the x-axis at (10, 0). The centre of the circle has coordinates

1. (7, 7)
2. (-10, 7)
- ✓ 3. (10, -7)
4. (7, -7)

5. A cylinder of radius 1 cm and height 1 cm is broken into three pieces. Which of the following MUST be true?

1. At least one piece has volume equal to  $1 \text{ cm}^3$ .
2. At least two pieces have equal volumes.
3. At least one piece has volume less than  $1 \text{ cm}^3$ .
- ✓ 4. At least one piece has volume greater than  $1 \text{ cm}^3$ .

6. One of the four – A, B, C and D – committed a crime. A said, "I did it." B said, "I didn't." C said, "B did it." D said, "A did it." Who is lying?

1. A
2. B
- ✓ 3. C
4. D

7. Consider the sequence of ordered sets of natural numbers:

{1}, {2, 3}, {4, 5, 6}, ...

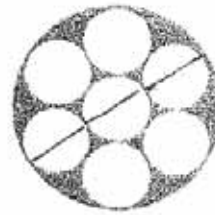
What is the last number in the 10<sup>th</sup> set?

1. 10
2. 19
- ✓ 3. 55
4. 67

8. If  $a + b + c + d + e = 10$  (all positive numbers), then the maximum value of  $a \times b \times c \times d \times e$  is

1. 12
- ✓ 2. 32
3. 48
4. 72

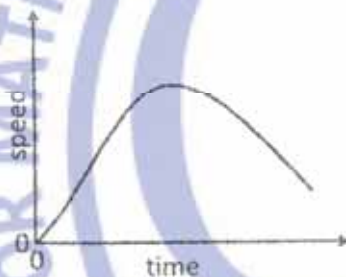
9. A farmer gives 7 full, 7 half-full and 7 empty bottles of honey to his three sons and asks them to share these among themselves such that each of them gets the same amount of honey and the same number of bottles. In how many ways can this be done? (bottles cannot be distinguished otherwise, they are sealed and cannot be broken).



1. 2
2.  $2\pi$
3.  $1/2$
4.  $\pi/2$

1. 0
2. 1
3. 2
4. 3

10. A car is moving along a straight track. Its speed is changing with time as shown.



12. What is the perimeter of the given figure, where adjacent sides are at right angles to each other?



1. 20 cm
2. 18 cm
3. 21 cm
4. cannot be determined

Which of the following statements is correct?

1. The speed is never zero.
2. The acceleration is zero once on the path.
3. The distance covered initially increases and then decreases.
4. The car comes back to its initial position once.

13. What does the diagram establish?



Note: The diagram is a circle inside a square.

11. A circle circumscribes identical, close-packed circles of unit diameter as shown. What is the total area of the shaded portion?

1.  $\pi > 3$
2.  $\pi \geq 2\sqrt{2}$
3.  $\pi < 4$
4.  $\pi$  is closer to 3 than to 4.

$$7\pi\left(\frac{1}{2}\right)^2 = 7$$

14. What is the next number in the following sequence?

39, 42, 46, 50, ...

- 1. 52
- 2. 53
- 3. 54
- 4. 55

15.  $(25 \div 5 + 3 - 2 \times 4) + (16 \times 4 - 3) =$

- 1. 61
- 2. 22
- 3. 41/24
- 4. 16

16. A student buys a book from an online shop at 20% discount. His friend buys another copy of the same book in a book fair for Rs.192 paying 20% less than his friend. What is the full price of the book?

- 1. Rs.275
- 2. Rs.300
- 3. Rs.320
- 4. Rs.392

17. How many nine-digit positive integers are there, the sum of squares of whose digits is 2?

- 1. 8
- 2. 9
- 3. 10
- 4. 11

18. 366 players participate in a knockout tournament. In each round of competing players pair up and play a match, the winner of each match moving to the next round. If at the end of a round there is an odd number of winners, the unpaired one moves to the next round without playing a match. What is the total number of matches played?

- 1. 366
- 2. 282
- 3. 365
- 4. 418

19. For real numbers  $x$  and  $y$ ,  $x^2 + (y-4)^2 = 0$ . Then the value of  $x^2 + y^2$  is

- 1. 0
- 2. 2
- 3.  $\sqrt{2}$
- 4. 4

20. There are 2 hills, A and B. Hill A is located N50°E of hill B. What will be the direction of hill B when observed from hill A? (N 30°E means 30° from north towards east.)

- 1. S 30°W
- 2. S 60°W
- 3. S 30°E
- 4. S 60°E

## MATHEMATICAL SCIENCES PAPER-I (PART-B)

41. Let  $\{x_n\}$  be a sequence of non-zero real numbers. Then
1. If  $x_n \rightarrow a$ , then  $a = \sup x_n$ .
  2. If  $\frac{x_{n+1}}{x_n} < 1 \quad \forall n$ , then  $x_n \rightarrow 0$ .
  3. If  $x_n < n \quad \forall n$ , then  $\{x_n\}$  diverges.
  4. If  $n \leq x_n \quad \forall n$ , then  $\{x_n\}$  diverges.
42. Let  $\{x_n\}$  and  $\{y_n\}$  be two sequences of real numbers such that  $x_n \leq y_n \leq x_{n+2}$ ,  $n = 1, 2, 3, \dots$
1.  $\{y_n\}$  is an bounded sequence.
  2.  $\{x_n\}$  is an increasing sequence.
  3.  $\{x_n\}$  and  $\{y_n\}$  converge together.
  4.  $\{y_n\}$  is an increasing sequence.
43. Let  $f: [0, 1] \rightarrow (0, \infty)$  be a continuous function. Suppose  $f(0) = 1$  and  $f(1) = 7$ . Then
1.  $f$  is uniformly continuous and is not onto.
  2.  $f$  is increasing and  $f([0, 1]) = [1, 7]$ .
  3.  $f$  is not uniformly continuous.
  4.  $f$  is not bounded.
44. Let  $f: [a, b] \rightarrow [c, d]$  be a monotone and bijective function. then
1.  $f$  is continuous, but  $f^{-1}$  need not be.
  2.  $f$  and  $f^{-1}$  are both continuous.
  3. If  $b - a > d - c$ , then  $f$  is a decreasing function.
  4.  $f$  is not uniformly continuous.
45. Let  $\sum_{n=1}^{\infty} x_n$  be a series of real numbers. Which of the following is true?
1. If  $\sum_{n=1}^{\infty} x_n$  is divergent, then  $\{x_n\}$  does not converge to 0.
  2. If  $\sum_{n=1}^{\infty} x_n$  is convergent, then  $\sum_{n=1}^{\infty} x_n$  is absolutely convergent.
  3. If  $\sum_{n=1}^{\infty} x_n$  is convergent, then  $x_n^2 \rightarrow 0$ , as  $n \rightarrow \infty$ .
  4. If  $x_n \rightarrow 0$ , then  $\sum_{n=1}^{\infty} x_n$  is convergent.

46. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be differentiable with  $0 < f'(x) < 1$  for all  $x$ . Then
1.  $f$  is increasing and  $f$  is bounded.
  2.  $f$  is increasing and  $f$  is Riemann integrable on  $\mathbb{R}$ .
  3.  $f$  is increasing and  $f$  is uniformly continuous.
  4.  $f$  is of bounded variation.
47. Let  $f_n: [0,1] \rightarrow \mathbb{R}$  be a sequence of differentiable functions. Assume that  $(f_n)$  converges uniformly on  $[0, 1]$  to a function  $f$ . Then
1.  $f$  is differentiable and Riemann integrable on  $[0, 1]$ .
  2.  $f$  is uniformly continuous and Riemann integrable on  $[0, 1]$ .
  3.  $f$  is continuous,  $f$  need not be differentiable on  $(0, 1)$  and need not be Riemann integrable on  $[0, 1]$ .
  4.  $f$  need not be uniformly continuous on  $[0, 1]$ .
48. Let, if possible,  $\alpha = \lim_{(x,y) \rightarrow (0,0)} \frac{\sin(x^2 + y^2)}{x^2 + y^2}$ ,  $\beta = \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{x^2 + y^2}$ . Then
1.  $\alpha$  exists but  $\beta$  does not.
  2.  $\alpha$  does not exist but  $\beta$  exists.
  3.  $\alpha, \beta$  do not exist.
  4. Both  $\alpha, \beta$  exist.
49. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a non-negative Lebesgue integrable function. Then
1.  $f$  is finite almost everywhere.
  2.  $f$  is a continuous function.
  3.  $f$  has at most countably many discontinuities.
  4.  $f^2$  is Lebesgue integrable.
50. Let  $S = \{(x, y) \in \mathbb{R}^2 : xy = 1\}$ . then
1.  $S$  is not connected but compact.
  2.  $S$  is neither connected nor compact.
  3.  $S$  is bounded but not connected.
  4.  $S$  is unbounded but connected.
51. Consider the linear space  $X = C[0, 1]$  with the norm  $\|f\| = \sup\{|f(t)| : 0 \leq t \leq 1\}$ .
- Let  $F = \left\{ f \in X : f\left(\frac{1}{2}\right) = 0 \right\}$  and  $G = \left\{ g \in X : g\left(\frac{1}{2}\right) \neq 0 \right\}$ .
- Then
1.  $F$  is not closed and  $G$  is open.
  2.  $F$  is closed but  $G$  is not open.
  3.  $F$  is not closed and  $G$  is not open.
  4.  $F$  is closed and  $G$  is open.

52. Let  $V$  be the vector space of all  $n \times n$  real matrices,  $A = [a_{ij}]$  such that  $a_{ij} = -a_{ji}$  for all  $i, j$ . Then the dimension of  $V$  is:

1.  $\frac{n^2 + n}{2}$ .
2.  $\frac{n^2 - n}{2}$ .
3.  $n^2 - n$ .
4.  $n$ .

53. Let  $n = mk$  where  $m$  and  $k$  are integers  $\geq 2$ . Let  $A = [a_{ij}]$  be a matrix given by  $a_{ij} = 1$  if for some  $r = 0, 1, \dots, m-1$ ,  $rk < i, j \leq (r+1)k$  and  $a_{ij} = 0$ , otherwise. Then the null space of  $A$  has dimension :

1.  $m(k-1)$ .
2.  $mk-1$ .
3.  $k(m-1)$ .
4. zero.

54. The set of all solutions to the system of equations :

$$\begin{aligned}(1-i)x_1 - ix_2 &= 0 \\ 2x_1 + (1-i)x_2 &= 0\end{aligned}$$

is given by:

1.  $(x_1, x_2) = (0, 0)$ .
2.  $(x_1, x_2) = (1, 1)$ .
3.  $(x_1, x_2) = c \left( 1, \cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4} \right)$  where  $c$  is any complex number.
4.  $(x_1, x_2) = c \left( \cos \frac{3\pi}{4}, i \sin \frac{3\pi}{4} \right)$  where  $c$  is any complex number.

55. Let  $A$  be an  $m \times n$  matrix where  $m < n$ . Consider the system of linear equations  $A\mathbf{x} = \mathbf{b}$  where  $\mathbf{b}$  is an  $n \times 1$  column vector and  $\mathbf{b} \neq \mathbf{0}$ . Which of the following is always true?

1. The system of equations has no solution.
2. The system of equations has a solution if and only if it has infinitely many solutions.
3. The system of equations has a unique solution.
4. The system of equations has at least one solution.

56. Let  $T$  be a normal operator on a complex inner product space. Then  $T$  is self-adjoint if and only if :

1. All eigenvalues of  $T$  are distinct.
2. All eigenvalues of  $T$  are real.
3.  $T$  has repeated eigenvalues.
4.  $T$  has at least one real eigenvalue.

57. A  $2 \times 2$  real matrix  $A$  is diagonalizable if and only if :

1.  $(\text{tr} A)^2 < 4 \text{Det } A$ .
2.  $(\text{tr } A)^2 > 4 \text{Det } A$ .
3.  $(\text{tr } A)^2 = 4 \text{Det } A$ .
4.  $\text{Tr } A = \text{Det } A$ .

58. Let  $A$  be a  $3 \times 3$  complex matrix such that  $A^3 = I$  (= the  $3 \times 3$  identity matrix). Then :

1.  $A$  is diagonalizable.
2.  $A$  is not diagonalizable.
3. The minimal polynomial of  $A$  has a repeated root.
4. All eigenvalues of  $A$  are real.

59. Let  $V$  be the real vector space of real polynomials of degree  $< 3$  and let  $T : V \rightarrow V$  be the linear transformation defined by  $P(t) \mapsto Q(t)$  where  $Q(t) = P(at + b)$ . Then the matrix of  $T$  with respect to the basis  $1, t, t^2$  of  $V$  is:

1. 
$$\begin{pmatrix} b & b & b^2 \\ 0 & a & 2ab \\ 0 & 0 & a^2 \end{pmatrix}.$$

2. 
$$\begin{pmatrix} a & a & a^2 \\ 0 & b & 2ab \\ 0 & 0 & b^2 \end{pmatrix}.$$

3. 
$$\begin{pmatrix} b & b & b^2 \\ a & a & 0 \\ 0 & b & a^2 \end{pmatrix}.$$

4. 
$$\begin{pmatrix} a & a & a^2 \\ b & b & 0 \\ 0 & a & b^2 \end{pmatrix}.$$

60. The minimal polynomial of the  $3 \times 3$  real matrix  $\begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{pmatrix}$  is:

1.  $(X - a)(X - b)$ .
2.  $(X - a)^2(X - b)$ .
3.  $(X - a)^2(X - b)^2$ .
4.  $(X - a)(X - b)^2$ .

61. The characteristic polynomial of the  $3 \times 3$  real matrix  $A = \begin{pmatrix} 0 & 0 & -c \\ 1 & 0 & -b \\ 0 & 1 & -a \end{pmatrix}$  is:

1.  $X^3 + aX^2 + bX + c$ .
2.  $(X - a)(X - b)(X - c)$ .
3.  $(X - 1)(X - abc)^2$ .
4.  $(X - 1)^2(X - abc)$ .

62. Let  $e_1, e_2, e_3$  denote the standard basis of  $\mathbb{R}^3$ . Then  $ae_1 + be_2 + ce_3, e_2, e_3$  is an orthonormal basis of  $\mathbb{R}^3$  if and only if

1.  $a \neq 0, a^2 + b^2 + c^2 = 1$ .
2.  $a = \pm 1, b = c = 0$ .
3.  $a = b = c = 1$ .
4.  $a = b = c$ .

63. Let  $E = \{z \in \mathbb{C} : e^z = i\}$ . Then  $E$  is :

1. a singleton.
2.  $E$  is a set of 4 elements.
3.  $E$  is an infinite set.
4.  $E$  is an infinite group under addition.

64. Suppose  $\{a_n\}$  is a sequence of complex numbers such that  $\sum_{n=0}^{\infty} a_n$  diverges. Then

the radius of convergence  $R$  of the power series  $\sum_{n=0}^{\infty} \frac{a_n}{2^n} (z - 1)^n$  satisfies :

1.  $R = 3$ .
2.  $R \leq 2$ .
3.  $R > 2$ .
4.  $R = \infty$ .

65. Let  $f, g$  be two entire functions. Suppose  $|f^2(z) + g^2(z)| = 1$ , then

1.  $f(z)f'(z) + g(z)g'(z) = 0$ .
2.  $f$  and  $g$  must be constant.
3.  $f$  and  $g$  are both bounded functions.
4.  $f$  and  $g$  have no zeros on the unit circle.

66. The integral  $\int_{|z|=2\pi} \frac{\sin z}{(z-\pi)^2}$  where the curve is taken anti-clockwise, equals :

1.  $-2\pi i$ .
2.  $2\pi i$ .
3.  $0$ .
3.  $4\pi i$ .

67. Suppose  $\{z_n\}$  is a sequence of complex numbers and  $\sum_{n=0}^{\infty} z_n$  converges.

Let  $f: \mathbb{C} \rightarrow \mathbb{C}$  be an entire function with  $f(z_n) = n, \forall n = 0, 1, 2, \dots$ . Then

1.  $f \equiv 0$ .
2.  $f$  is unbounded.
3. no such function exists.
4.  $f$  has no zeros.

68. Let  $f(z) = \cos z$  and  $g(z) = \cos z^2$ , for  $z \in \mathbb{C}$ . Then

1.  $f$  and  $g$  are both bounded on  $\mathbb{C}$ .
2.  $f$  is bounded, but  $g$  is not bounded on  $\mathbb{C}$ .
3.  $g$  is bounded, but  $f$  is not bounded on  $\mathbb{C}$ .
4.  $f$  and  $g$  are both bounded on the  $x$ -axis.

69. Let  $f$  be an analytic function and let  $f(z) = \sum_{n=0}^{\infty} a_n (z-2)^{2n}$  be its Taylor series in some disc. Then

1.  $f^{(n)}(0) = (2n)!a_n$
2.  $f^{(n)}(2) = n!a_n$
3.  $f^{(2n)}(2) = (2n)!a_n$
4.  $f^{(2n)}(2) = n!a_n$

70. The signature of the permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & \dots & n \\ n & n-1 & n-2 & \dots & 1 \end{pmatrix} \text{ is}$$

1.  $(-1)^{\binom{n}{2}}$ .
  2.  $(-1)^n$ .
  3.  $(-1)^{n+1}$ .
  4.  $(-1)^{n-1}$ .
71. Let  $\alpha$  be a permutation written as a product of disjoint cycles,  $k$  of which are cycles of odd size and  $m$  of which are cycles of even size, where  $4 \leq k \leq 6$  and  $6 \leq m \leq 8$ . It is also known that  $\alpha$  is an odd permutation. Then which one of the following is true?
1.  $k = 4$  and  $m = 6$ .
  2.  $m = 7$ .
  3.  $k = 6$ .
  4.  $m = 8$ .
72. Let  $p, q$  be two distinct prime numbers. then  $p^{q-1} + q^{p-1}$  is congruent to
1.  $1 \pmod{pq}$ .
  2.  $2 \pmod{pq}$ .
  3.  $p-1 \pmod{pq}$ .
  4.  $q-1 \pmod{pq}$ .
73. What is the total number of groups (upto isomorphism) of order 8?
1. only one.
  2. 3.
  3. 5.
  4. 6.
74. Which ones of the following three statements are correct?
- (A) Every group of order 15 is cyclic.  
 (B) Every group of order 21 is cyclic.  
 (C) Every group of order 35 is cyclic.
1. (A) and (C).
  2. (B) and (C).
  3. (A) and (B).
  4. (B) only.

75. Let  $p$  be a prime number and consider the natural action of the group  $GL_2(\mathbb{F}_p)$  on  $\mathbb{F}_p \times \mathbb{F}_p$ . Then the index of the isotropy subgroup at  $(1, 1)$  is
1.  $p^2 - 1$ .
  2.  $p(p-1)$ .
  3.  $p-1$ .
  4.  $p^2$ .
76. The quadratic polynomial  $X^2 + bX + c$  is irreducible over the finite field  $\mathbb{F}_5$  if and only if
1.  $b^2 - 4c = 1$ .
  2.  $b^2 - 4c = 4$ .
  3. either  $b^2 - 4c = 2$  or  $b^2 - 4c = 3$ .
  4. either  $b^2 - 4c = 1$  or  $b^2 - 4c = 4$ .
77. Let  $K$  denote a proper subfield of the field  $F = GF(2^{12})$  a finite field with  $2^{12}$  elements. Then the number of elements of  $K$  must be equal to
1.  $2^m$  where  $m = 1, 2, 3, 4$  or  $6$ .
  2.  $2^m$  where  $m = 1, 2, \dots, 11$ .
  3.  $2^{12}$ .
  4.  $2^m$  where  $m$  and  $12$  are coprime to each other.
78. The general and singular solutions of the differential equation  $y = \frac{9}{2}x p^{-1} + \frac{1}{2}px$ , where  $p = \frac{dy}{dx}$ , are given by
1.  $2cy - x^2 - 9c^2 = 0, 3y = 2x$ .
  2.  $2cy - x^2 + 9c^2 = 0, y = \pm 3x$ .
  3.  $2cy + x^2 + 9c^2 = 0, y = \pm 3x$ .
  4.  $2cy + x^2 + 9c^2 = 0, 3y = 4x$ .
79. A homogenous linear differential equation with real constant coefficients, which has  $y = xe^{-3x} \cos 2x + e^{-3x} \sin 2x$ , as one of its solutions, is given by:
1.  $(D^2 + 6D + 13)y = 0$ .
  2.  $(D^2 - 6D + 13)y = 0$ .
  3.  $(D^2 - 6D + 13)^2 y = 0$ .
  4.  $(D^2 + 6D + 13)^2 y = 0$ .

80. The particular integral  $y_p(x)$  of the differential equation

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = \frac{1}{x+1}, \quad x > 0$$

is given by

$$y_p(x) = x v_1(x) + \frac{1}{x} v_2(x)$$

where  $v_1(x)$  and  $v_2(x)$  are given by

1.  $x v_1'(x) - \frac{1}{x^2} v_2'(x) = 0, \quad v_1'(x) - \frac{1}{x^2} v_2'(x) = \frac{1}{x+1}.$
2.  $x v_1'(x) + \frac{1}{x^2} v_2'(x) = 0, \quad v_1'(x) - \frac{1}{x^2} v_2'(x) = \frac{1}{x+1}.$
3.  $x v_1'(x) - \frac{1}{x^2} v_2'(x) = 0, \quad v_1'(x) + \frac{1}{x^2} v_2'(x) = \frac{1}{x+1}.$
4.  $x v_1'(x) + \frac{1}{x^2} v_2'(x) = 0, \quad v_1'(x) + \frac{1}{x^2} v_2'(x) = \frac{1}{x+1}.$

81. The boundary value problem  $y'' + \lambda y = 0, y(0) = 0, y(\pi) + k y'(\pi) = 0$ , is self-adjoint

1. only for  $k \in \{0, 1\}$ .
2. for all  $k \in (-\infty, \infty)$ .
3. only for  $k \in [0, 1]$ .
4. only for  $k \in (-\infty, 1) \cup (1, \infty)$ .

82. The general integral of  $z(xp - yq) = y^2 - x^2$  is

1.  $z^2 - x^2 + y^2 + f(xy).$
2.  $z^2 = x^2 - y^2 + f(xy).$
3.  $z^2 = -x^2 - y^2 + f(xy).$
4.  $z^2 = y^2 - x^2 + f(xy).$

83. A singular solution of the partial differential equation  $z + xp - x^2y q^2 - x^3pq = 0$  is

1.  $z = \frac{x^2}{y}$ .

2.  $z = \frac{x}{y^2}$ .

3.  $z = \frac{y}{x^2}$ .

4.  $z = \frac{y^2}{x}$ .

84. The characteristics of the partial differential equation

$$36 \frac{\partial^2 z}{\partial x^2} - y^{14} \frac{\partial^2 z}{\partial y^2} - y^{13} \frac{\partial z}{\partial y} = 0, \text{ are given by}$$

1.  $x + \frac{1}{y^6} = c_1, x - \frac{1}{y^6} = c_2$ .

2.  $x + \frac{36}{y^6} = c_1, x - \frac{36}{y^6} = c_2$ .

3.  $6x + \frac{7}{y^6} = c_1, 6x - \frac{7}{y^6} = c_2$ .

4.  $6x + \frac{7}{y^8} = c_1, 6x - \frac{7}{y^8} = c_2$ .

85. The Lagrange interpolation polynomial through  $(1, 10), (2, -2), (3, 8)$ , is

1.  $11x^2 - 45x + 38$ .

2.  $11x^2 - 45x + 36$ .

3.  $11x^2 - 45x + 30$ .

4.  $11x^2 - 45x + 44$ .

86. Newton's method for finding the positive square root of  $a > 0$  gives, assuming  $x_0 > 0$ ,  $x_0 \neq \sqrt{a}$ ,

1.  $x_{n+1} = \frac{x_n}{2} + \frac{a}{x_n}.$

2.  $x_{n+1} = \frac{1}{2} \left( x_n + \frac{a}{x_n} \right).$

3.  $x_{n+1} = \frac{1}{\sqrt{2}} \left( x_n - \frac{a}{x_n} \right).$

4.  $x_{n+1} = \frac{1}{\sqrt{2}} \left( x_n + \frac{a}{x_n} \right).$

87. The extremal problem

$$J[y(x)] = \int_0^{\pi} \{ (y')^2 - y^2 \} dx$$

$$y(0)=1, y(\pi)=\lambda, \text{ has}$$

1. a unique extremal if  $\lambda = 1$ .
2. infinitely many extremals if  $\lambda = 1$ .
3. a unique extremal if  $\lambda = -1$ .
4. infinitely many extremal if  $\lambda = -1$ .

88. The functional

$$J[y] = \int_0^1 e^x (y^2 + \frac{1}{2} y'^2) dx; y(0)=1, y(1)=e$$

attains

1. A weak, but not a strong minimum on  $e^x$ .
2. A strong minimum on  $e^x$ .
3. A weak, but not a strong maximum on  $e^x$ .
4. A strong maximum on  $e^x$ .

89. A solution of the integral equation

$$\int_0^x e^{x-t} \phi(t) dt = \sinh x, \text{ is}$$

1.  $\phi(x) = e^{-x}.$
2.  $\phi(x) = e^x.$
3.  $\phi(x) = \sinh x.$
4.  $\phi(x) = \cosh x.$

90. If  $\bar{\varphi}(p)$  denotes the Laplace transform of  $\varphi(x)$  then for the integral equation of convolution type

$$\varphi(x) = 1 + 2 \int_0^x \cos(x-t) \varphi(t) dt,$$

$\bar{\varphi}(p)$  is given by

1.  $\frac{p^2 + 1}{(p-1)^2}.$
2.  $\frac{p^2 + 1}{(p+1)^2}.$
3.  $\frac{(p^2 + 1)}{p(p-1)^2}.$
4.  $\frac{p^2 + 1}{p(p+1)^2}.$

91. The Lagrangian of a dynamical system is  $L = \dot{q}_1^2 + \dot{q}_2^2 + k_1 q_1^2$ , then the Hamiltonian is given by

1.  $H = p_1^2 + p_2^2 - k_1 q_1^2.$
2.  $H = \frac{1}{4}(p_1^2 + p_2^2) + k_1 q_1^2.$
3.  $H = p_1^2 + p_2^2 + k_1 q_1^2.$
4.  $H = \frac{1}{4}(p_1^2 + p_2^2) - k_1 q_1^2.$

92. The kinetic energy T and potential energy V of a dynamical system are given respectively, under usual notations, by

$$T = \frac{1}{2} [A(\dot{\theta}^2 + \dot{\phi}^2 \sin^2 \theta) + B(\dot{\phi} \cos \theta + \dot{\phi}^2)]$$

and  $V = Mgl \cos \theta$ . The generalized momentum  $p_\phi$  is

1.  $p_\phi = 2B\dot{\phi} \cos \theta + 2\dot{\phi}^2.$
2.  $p_\phi = \frac{B}{2}(\dot{\phi} \cos \theta + \dot{\phi}^2).$
3.  $p_\phi = B(\dot{\phi} \cos \theta + \dot{\phi}^2).$
4.  $p_\phi = B(\dot{\phi} \cos \theta + \dot{\phi}).$

93. Consider repeated tosses of a coin with probability  $p$  for head in any toss. Let  $NB(k, p)$  be the random variable denoting the number of tails before the  $k^{\text{th}}$  head.

Then  $P(NB(10, p) = j \mid 3^{\text{rd}}$  head occurred in  $15^{\text{th}}$  toss) is equal to

1.  $P(NB(7, p) = j - 15)$ , for  $j = 15, 16, \dots$
2.  $P(NB(7, p) = j - 12)$ , for  $j = 12, 13, \dots$
3.  $P(NB(10, p) = j - 15)$ , for  $j = 15, 16, \dots$
4.  $P(NB(10, p) = j - 12)$ , for  $j = 12, 13, \dots$

94. Suppose  $X$  and  $Y$  are standard normal random variables. Then which of the following statements is correct?

1.  $(X, Y)$  has a bivariate normal distribution.
2.  $\text{Cov}(X, Y) = 0$ .
3. The given information does not determine the joint distribution of  $X$  and  $Y$ .
4.  $X + Y$  is normal.

95. Let  $F$  be the distribution function of a strictly positive random variable with finite expectation  $\mu$ . Define

$$G(x) = \begin{cases} \frac{1}{\mu} \int_0^x (1 - F(y)) dy, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

Which of the following statements is correct?

1.  $G$  is a decreasing function.
2.  $G$  is a probability density function.
3.  $G(x) \rightarrow +\infty$  as  $x \rightarrow +\infty$ .
4.  $G$  is a distribution function.

96. Let  $X_1, X_2, \dots$  be an irreducible Markov chain on the state space  $\{1, 2, \dots\}$ . Then  $P(X_n = 5 \text{ for infinitely many } n)$  can equal

1. Only 0 or 1.
2. Only 0.
3. Any number in  $[0, 1]$ .
4. Only 1.

97.  $X_1, X_2, \dots, X_n$  is a random sample from a normal population with mean zero and variance  $\sigma^2$ . Let  $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$ . Then the distribution of  $T = \sum_{i=1}^{n-1} (X_i - \bar{X})$  is

1.  $t_{n-1}$
2.  $N(0, (n-1)\sigma^2)$
3.  $N(0, \frac{n+1}{n}\sigma^2)$
4.  $N(0, \frac{n-1}{n}\sigma^2)$

98. Let  $X_1, X_2, \dots, X_n$  be independent exponential random variables with parameters  $\lambda_1, \lambda_2, \dots, \lambda_n$  respectively. Let  $Y = \min(X_1, \dots, X_n)$ . Then  $Y$  has an exponential distribution with parameter

1.  $\sum_{i=1}^n \lambda_i$
2.  $\prod_{i=1}^n \lambda_i$
3.  $\min\{\lambda_1, \dots, \lambda_n\}$
4.  $\max\{\lambda_1, \dots, \lambda_n\}$

99. Suppose  $x_1, x_2, \dots, x_n$  are  $n$  observations on a variable  $X$ . Then the value of  $A$  which minimizes  $\sum_{i=1}^n (x_i - A)^2$  is

1. median of  $x_1, x_2, \dots, x_n$
2. mode of  $x_1, x_2, \dots, x_n$
3. mean of  $x_1, x_2, \dots, x_n$
4.  $\frac{\min(x_1, \dots, x_n) + \max(x_1, \dots, x_n)}{2}$

100. Suppose  $X_1, X_2, \dots, X_n$  are i.i.d. with density function  $f(x) = \frac{\theta}{x^2}, \theta < x, \theta > 0$ . Then

1.  $\sum_{i=1}^n \frac{1}{X_i^2}$  is sufficient for  $\theta$
2.  $\min_{1 \leq i \leq n} X_i$  is sufficient for  $\theta$ .
3.  $\prod_{i=1}^n \frac{1}{X_i^2}$  is sufficient for  $\theta$
4.  $\left(\max_{1 \leq i \leq n} X_i, \min_{1 \leq i \leq n} X_i\right)$  is not sufficient for  $\theta$ .

101. Suppose  $X$  is a random variable with density function  $f(x)$ .  
To test  $H_0 : f(x) = 1, 0 < x < 1$ , vs  $H_1 : f(x) = 2x, 0 < x < 1$ , the UMP test at level  $\alpha = 0.05$
1. Does not exist
  2. Rejects  $H_0$  if  $X > 0.95$
  3. Rejects  $H_0$  if  $X > 0.05$
  4. Rejects  $H_0$  for  $X < C_1$  or  $X > C_2$  where  $C_1, C_2$  have to be determined.
102. Suppose the distribution of  $X$  is known to be one of the following:
- $$f_1(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, -\infty < x < \infty;$$
- $$f_2(x) = \frac{1}{2} e^{-|x|}, -\infty < x < \infty;$$
- $$f_3(x) = \frac{1}{4}, -2 < x < 2.$$
- If  $X = 0$  is observed, then the maximum likelihood estimate of the distribution of  $X$  is
1.  $f_1(x)$
  2.  $f_2(x)$
  3.  $f_3(x)$
  4. Does not exist.
103. Suppose  $X_i, i = 1, 2, \dots, n$ , are independently and identically distributed random variables with common distribution function  $F(\cdot)$ . Suppose  $F(\cdot)$  is absolutely continuous and the hypothesis to be tested is  $p^{\text{th}} (0 < p < 1/2)$  quantile is  $\xi_0$ . An appropriate test is
1. Sign Test
  2. Mann-Whitney Wilcoxon rank sum test
  3. Wilcoxon Signed rank test
  4. Kolmogorov Smirnov test

104. Suppose  $Y \sim N(\theta, \sigma^2)$  and suppose the prior distribution on  $\theta$  is  $N(\mu, \tau^2)$ . The posterior distribution of  $\theta$  is also  $N\left(\frac{\tau^2}{\tau^2 + \sigma^2}y + \frac{\sigma^2}{\tau^2 + \sigma^2}\mu, \frac{\sigma^2 \tau^2}{\tau^2 + \sigma^2}\right)$

The Bayes' estimator of  $\theta$  under squared error loss is given by

1.  $\frac{\tau^2}{\tau^2 + \sigma^2} y$
  2.  $\frac{\tau^2 y}{\tau^2 + \sigma^2}$
  3.  $\frac{\tau^2}{\tau^2 + \sigma^2} y + \frac{\sigma^2}{\tau^2 + \sigma^2} \mu$
  4.  $y$ .
105. Consider the model  
 $y_{ij} = \mu + \theta(i-1) + \beta(2-j) + \varepsilon_{ij}, \quad i = 1, 2; j = 1, 2,$   
 where  $y_{ij}$  is the observation under  $i^{\text{th}}$  treatment and  $j^{\text{th}}$  block,  $\mu$  is the general effect,  $\theta$  and  $\beta$  are treatment and block parameters respectively and  $\varepsilon_{ij}$  are random errors with mean 0 and common variance  $\sigma^2$ . Then
1.  $\mu, \theta$  and  $\beta$  are all estimable
  2.  $\theta$  and  $\beta$  are estimable,  $\mu$  is not estimable
  3.  $\mu$  and  $\theta$  are estimable,  $\beta$  is not estimable
  4.  $\mu$  and  $\beta$  are estimable,  $\theta$  is not estimable
106. Consider a multiple linear regression model  $\underline{y} = X \underline{\beta} + \underline{\varepsilon}$   
 where  $\underline{y}$  is a  $n \times 1$  vector of response variables,  $X$  is a  $n \times p$  regression matrix,  $\underline{\beta}$  is a  $p \times 1$  vector of unknown parameters and  $\underline{\varepsilon}$  is a  $n \times 1$  vector of uncorrelated random variables with mean 0 and common variance  $\sigma^2$ . Let  $\hat{\underline{y}}$  be the vector of least squares fitted values of  $\underline{y}$  and  $\underline{e} = (e_1, \dots, e_n)^T$  be the vector of residuals. Then

1.  $\sum_{i=1}^n e_i = 0$  always
2.  $\sum_{i=1}^n e_i = 0$  if one column of  $X$  is  $(1, \dots, 1)^T$
3.  $\sum_{i=1}^n e_i = 0$  only if one column of  $X$  is  $(1, \dots, 1)^T$
4. nothing can be said about  $\sum_{i=1}^n e_i$

107. Suppose  $X_{0/p \times 1} \sim N_p(0, \Sigma)$  where

$$\Sigma = \begin{pmatrix} 1 & -1/2 & 0 & L & 0 \\ -1/2 & 1 & 0 & L & 0 \\ 0 & 0 & & & \\ M & M & & \Sigma_{22} & \\ 0 & 0 & & & \end{pmatrix}$$

and  $\Sigma_{22}$  is positive definite. Then

$P(X_1 - X_2 < 0, X_1 + X_2 \neq 0 \mid X_P > 0)$  is equal to

1.  $1/8$
2.  $1/4$
3.  $1/2$
4.  $1$

108. Suppose the variance-covariance matrix of a random vector  $\underline{X}_{(3 \times 1)}$  is

$$\Sigma = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 8 & 2 \\ 0 & 2 & 8 \end{pmatrix}.$$

The percentage of variation explained by the first principal component is

1. 50
2. 45
3. 60
4. 40

109. A population consists of 10 students. The marks obtained by one student is 10 less than the average of the marks obtained by the remaining 9 students. Then the variance of the population of marks ( $\sigma^2$ ) will always satisfy

1.  $\sigma^2 \geq 10$
2.  $\sigma^2 = 10$
3.  $\sigma^2 \leq 10$
4.  $\sigma^2 \geq 9$

110. For what value of  $\lambda$ , the following will be the incidence matrix of a BIBD?

$$N = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & \lambda \\ 0 & 1 & 1 \end{pmatrix}$$

1.  $\lambda = 0$
2.  $\lambda = 1$
3.  $\lambda = 4$
4.  $\lambda = 3$

111. With reference to a  $2^2$  – factorial experiment, consider the factorial effects A, B and AB. Then the estimates of

1. Only A and B are orthogonal
2. Only A and C are orthogonal
3. Only B and C are orthogonal
4. A, B and C are orthogonal

112. Let X be a r.v. denoting failure time of a component. Failure rate of the component is constant if and only if p.d.f. of X is

1. exponential
2. negative binomial
3. weibull
4. normal

113. Consider the problem

$$\begin{aligned} \max \quad & 6x_1 - 2x_2 \\ \text{subject to} \quad & x_1 - x_2 \leq 1 \\ & 3x_1 - x_2 \leq 6 \\ & x_1, x_2 \geq 0 \end{aligned}$$

This problem has

1. unbounded solution space but unique optimal solution with finite optimum objective value
2. unbounded solution space as well as unbounded objective value
3. no feasible solution
4. unbounded solution space but infinite optimal solutions with finite optimum objective value

114. Consider an M/M/1/K queuing system in which at most K customers are allowed in the system with parameters  $\lambda$  and  $\mu$ , respectively ( $\rho = \lambda / \mu$ ). The expected steady state number of customers in the queueing system is K/2 for

1.  $\rho = 1$
2.  $\rho < 1$
3.  $\rho > 1$
4. any  $\rho$

115. Consider the system of equations  
 $P_1x_1 + P_2x_2 + P_3x_3 + P_4x_4 = b$ , where

$$P_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, P_2 = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}, P_3 = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}, P_4 = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}, b = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}.$$

The following vector combination does not form a basis:

1.  $(P_1, P_2, P_3)$
2.  $(P_1, P_2, P_4)$
3.  $(P_2, P_3, P_4)$
4.  $(P_1, P_3, P_4)$ .