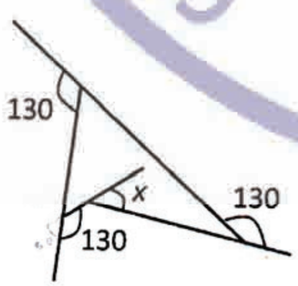
PART 'A'

1. A pyramid shaped toy is made by tightly placing cubic blocks of $1 \times 1 \times 1$ cm^3. The base of the toy is a square 4×4 cm^2. The width of each step is 0.5 cm. How many blocks are required to make the toy?
   1. 30 2. 34
   3. 36 4. 40

2. Of three persons A, B and C, one always lies while the others always speak the truth. C asked A, "Do you always speak the truth, yes or no?" He said something that C could not hear. So, C asked B, "what did A say?"
   B replied, "A said No".
   So, who is the liar?
   1. A 2. B
   3. C 4. cannot be determined

3. Two plane mirrors facing each other are kept at 60° to each other. A point is located on the angle bisector. The number of images of the point is
   1. 6 2. 3
   3. 5 4. Infinite

4. What is angle x in the schematic diagram given below ?



   1. 60 2. 50
   3. 40 4. 30

5. Consider 3 parallel strips of 10 m width running around the Earth, parallel to the equator; A_1 at the Equator, A_2 at the Tropic of Cancer and A_3 at the Arctic Circle. The order of the areas of the strips is
   1. $A_1 < A_2 < A_3$ 2. $A_1 = A_2 > A_3$
   3. $A_1 > A_2 = A_3$ 4. $A_1 > A_2 > A_3$

6. A 3 m long car goes past a 4 m long truck at rest on the road. The speed of the car is 7 m/s. The time taken to go past is
   1. 4/7 s 2. 1 s
   3. 7/4 s 4. 10/7 s

7. Let m and n be two positive integers such that
   $m + n + mn = 118$
   Then the value of $m + n$ is
   1. not uniquely determined
   2. 18
   3. 20
   4. 22

8. I bought a shirt at 10% discount and sold it to a friend at a loss of 10%. If the friend paid me Rs. 729.00 for the shirt, what was the undiscounted price of the shirt?
   1. Rs. 900 2. Rs. 800
   3. Rs. 1000 4. Rs. 911.25

9. Suppose
   (1) $x = 4$
   (2) Then $x - 4 = x^2 - 4^2$ (as both sides are zero)
   (3) Therefore $(x-4) = (x-4)(x+4)$
   Cancelling $(x-4)$ from both sides
   (4) $1 = (x+4)$
   (5) Then $x = -3$
   Which is the wrong step?
   1. 1 to 2 2. 2 to 3
   3. 3 to 4 4. 4 to 5

10. From a group of 40 players, a cricket team of 11 players is chosen. Then, one of the eleven is chosen as the captain of the team. The total number of ways this can be done is

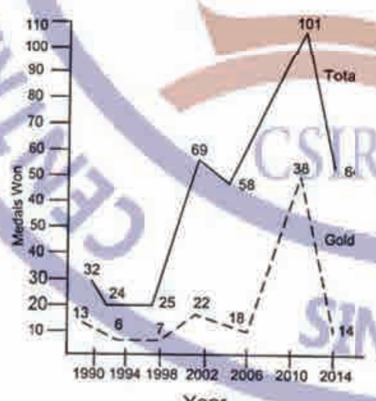
[$\binom{m}{n}$ below means the number of ways n objects can be chosen from m objects]

1. $\binom{40}{11}$
2. $11\binom{40}{11}$
3. $29\binom{40}{11}$
4. $\binom{39}{10}$

11. Information in DNA is in the form of sequence of 4 bases namely A, T, G and C. The proportion of G is the same as that of C, and that of A is the same as that of T. Which of the following strands of DNA will potentially have maximum diversity (i.e., maximum information content per base)?

1. length 1000 bases with 10% G
2. length 2000 bases with 10% A
3. length 2000 bases with 40% T
4. length 1000 bases with 25% C

12.



Based on the graph, which of the following statements is NOT true?

1. Number of gold medals increased whenever total number of medals increased
2. Percentage increase in gold medals in 2010 over 2006 is more than the corresponding increase in total medals
3. Every time non-gold medals together account for more than 50% of the total medals
4. Percentage increase in gold medals in 2010 over 2006 is more than the corresponding increase in 2002 over 1998

13. How many non-negative integers less than 10,000 are there such that the sum of the digits of the number is divisible by three?

1. 1112
2. 2213
3. 2223
4. 3334

14. In each of the following groups of words is a hidden number, based on which you should arrange them in ascending order. Pick the right answer:

A. Tinsel event
B. Man in England
C. Good height
D. Last encounter

1. A, B, C, D
2. C, B, D, A
3. A, C, D, B
4. C, D, B, A

15. Starting from a point A you fly one mile south, then one mile east, then one mile north which brings you back to point A. Point A is NOT the north pole. Which of the following MUST be true?

1. You are in the Northern Hemisphere
2. You are in the Eastern Hemisphere
3. You are in the Western Hemisphere
4. You are in the Southern Hemisphere

16. A single celled spherical organism contains 70% water by volume. If it loses 10% of its water content, how much would its surface area change by approximately?
    1. 3%
    2. 5%
    3. 6%
    4. 7%

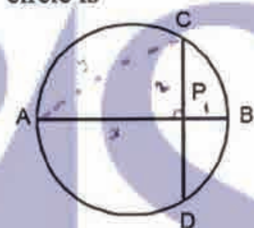
17. Jar W contains 40 white marbles and jar B contains 40 black marbles. Ten black marbles from B are transferred to W and mixed thoroughly. Now, ten randomly selected marbles from W are put back in Jar B to make 40 marbles in each jar. The number of black marbles in W
    1. would be equal to the number of white marbles in B
    2. would be more than the number of white marbles in B
    3. would be less than the number of white marbles in B
    4. cannot be determined from the information given

18.

Two ants, initially at diametrically opposite points A and B on a circular ring of radius R, start crawling towards each other. The speed of the one at A is half of that of the one at B. The point at which they meet is at a straight line distance of
1. R from A
2. $\frac{3R}{2}$ from A
3. R from B
4. $\frac{3R}{2}$ from B

19. If $aN \Rightarrow S$
$cF \Rightarrow I$
$gH \Rightarrow M$
then $nS \Rightarrow ?$
1. T
2. A
3. L
4. K

20. AB is the diameter of a circle. The chord CD is perpendicular to AB intersecting it at P. If CP = 2 and PB = 1, the radius of the circle is



1. 1
2. 2.5
3. 2
4. 5

PART 'B'

UNIT - 1

21. Given a 4×4 real matrix A, let $T: \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation defined by $Tv = Av$, where we think of $\mathbb{R}^4$ as the set of real 4×1 matrices. For which choices of A given below, do Image(T) and Image(T^2) have respective dimensions 2 and 1? ($*$ denotes a nonzero entry)

1. $A = \begin{bmatrix} 0 & 0 & * & * \\ 0 & 0 & * & * \\ 0 & 0 & 0 & * \\ 0 & 0 & 0 & 0 \end{bmatrix}$
2. $A = \begin{bmatrix} 0 & 0 & * & 0 \\ 0 & 0 & * & 0 \\ 0 & 0 & 0 & * \\ 0 & 0 & 0 & * \end{bmatrix}$
3. $A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & * \\ 0 & 0 & * & 0 \end{bmatrix}$
4. $A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & * & * \\ 0 & 0 & * & * \end{bmatrix}$

22. The sum of the series $\frac{1}{1!} + \frac{1+2}{2!} + \frac{1+2+3}{3!} + \cdots$ equals

1. e
2. $\frac{e}{2}$
3. $\frac{3e}{2}$
4. $1 + \frac{e}{2}$

23. Which of the following is a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$?

a. $f\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ x + y \end{pmatrix}$

b. $g\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} xy \\ x + y \end{pmatrix}$

c. $h\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z - x \\ x + y \end{pmatrix}$

1. only f.
2. only g.
3. only h.
4. all the transformations f, g and h.

24. Let T be a 4×4 real matrix such that $T^4 = 0$. Let $k_i := \dim Ker\, T^i$ for $1 \le i \le 4$. Which of the following is NOT a possibility for the sequence $k_1 \le k_2 \le k_3 \le k_4$?

1. $3 \le 4 \le 4 \le 4$.
2. $1 \le 3 \le 4 \le 4$.
3. $2 \le 4 \le 4 \le 4$.
4. $2 \le 3 \le 4 \le 4$.

25. The limit

$$\lim_{x \to 0} \frac{1}{x} \int_x^{2x} e^{-t^2}\, dt$$

1. does not exist.
2. is infinite.
3. exists and equals 1.
4. exists and equals 0.

26. Let A, B be $n \times n$ matrices. Which of the following equals trace(A^2B^2)?

1. $(\text{trace}(AB))^2$
2. trace(AB^2A)
3. trace($(AB)^2$)
4. trace($BABA$)

27. Let A be an $m \times n$ matrix of rank n with real entries. Choose the correct statement.

1. $Ax = b$ has a solution for any b.
2. $Ax = 0$ does not have a solution.
3. If $Ax = b$ has a solution, then it is unique.
4. $y'A = 0$ for some nonzero y, where y' denotes the transpose of the vector y.

28. Let V be the space of twice differentiable functions on $\mathbb{R}$ satisfying
$f'' - 2f' + f = 0$.
Define $T: V \to \mathbb{R}^2$ by $T(f) = (f'(0), f(0))$.
Then T is
1. one-to-one and onto.
2. one-to-one but not onto.
3. onto but not one-to-one.
4. neither one-to-one nor onto.

29. Which of the following subsets of $\mathbb{R}^n$ is compact (with respect to the usual topology of $\mathbb{R}^n$)?
1. $\{(x_1, x_2, ..., x_n): |x_i| < 1, 1 \leq i \leq n\}$
2. $\{(x_1, x_2, ..., x_n): x_1 + x_2 + \cdots + x_n = 0\}$
3. $\{(x_1, x_2, ..., x_n): x_i \geq 0, 1 \leq i \leq n\}$
4. $\{(x_1, x_2, ..., x_n): 1 \leq x_i \leq 2^i, 1 \leq i \leq n\}$

30. Let $f: X \to X$ such that $f(f(x)) = x$ for all $x \in X$. Then
1. f is one-to-one and onto.
2. f is one-to-one, but not onto.
3. f is onto but not one-to-one.
4. f need not be either one-to-one or onto.

31. A polynomial of odd degree with real coefficients must have
1. at least one real root.
2. no real root.
3. only real roots.
4. at least one root which is not real.

32. The row space of a 20×50 matrix A has dimension 13. What is the dimension of the space of solutions of $Ax = 0$?

1. 7
2. 13
3. 33
4. 37

UNIT - 2

33. Let, for each $n \geq 1$, C_n be the open disc in $\mathbb{R}^2$, with centre at the point $(n, 0)$ and radius equal to n. Then $C = \bigcup_{n \geq 1} C_n$ is
1. $\{(x, y) \in \mathbb{R}^2: x > 0 \text{ and } |y| < x\}$.
2. $\{(x, y) \in \mathbb{R}^2: x > 0 \text{ and } |y| < 2x\}$.
3. $\{(x, y) \in \mathbb{R}^2: x > 0 \text{ and } |y| < 3x\}$.
4. $\{(x, y) \in \mathbb{R}^2: x > 0\}$.

34.
$$\int_{|z+1|=2} \frac{z^2}{4 - z^2} dz =$$
1. 0.
2. $-2\pi i$.
3. $2\pi i$.
4. 1.

35. Let f be a real valued harmonic function on $\mathbb{C}$, that is, f satisfies the equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$. Define the functions
$$g = \frac{\partial f}{\partial x} - i\frac{\partial f}{\partial y}$$
$$h = \frac{\partial f}{\partial x} + i\frac{\partial f}{\partial y}$$
Then
1. g and h are both holomorphic functions.
2. g is holomorphic, but h need not be holomorphic.
3. h is holomorphic, but g need not be holomorphic.
4. both g and h are identically equal to the zero function.

36. Let D be the set of tuples $(w_1, \dots, w_{10})$, where $w_i \epsilon \{1,2,3\}$, $1 \le i \le 10$ and $w_i + w_{i+1}$ is an even number for each i with $1 \le i \le 9$.
Then the number of elements in D is

1. $2^{11} + 1$.
2. $2^{10} + 1$.
3. $3^{10} + 1$.
4. $3^{11} + 1$.

37. Let R be the ring $\mathbb{Z}[x]/((x^2 + x + 1)(x^3 + x + 1))$ and I be the ideal generated by 2 in R. What is the cardinality of the ring R?

1. 27.
2. 32.
3. 64.
4. Infinite.

38. Up to isomorphism, the number of abelian groups of order 108 is:

1. 12.
2. 9.
3. 6.
4. 5.

39. The number of subfields of a field of cardinality 2^{100} is

1. 2.
2. 4.
3. 9.
4. 100.

40. How many elements does the set

$\{z \in \mathbb{C} \mid z^{60} = -1,\ z^k \neq -1 \text{ for } 0 < k < 60\}$ have?

1. 24.
2. 30.
3. 32.
4. 45.

UNIT - 3

41. The integral equation
$y(x) = \lambda \int_0^1 (3x - 2)t\, y(t)dt$, with λ as a parameter, has

1. only one characteristic number
2. two characteristic numbers
3. more than two characteristic numbers
4. no characteristic number

42. Let $f: \mathbb{R} \to \mathbb{R}$ be a polynomial of the form $f(x) = a_0 + a_1 x + a_2 x^2$ with $a_0, a_1, a_2 \in \mathbb{R}$ and $a_2 \neq 0$. If

$$E_1 = \int_{-1}^{1} f(x)dx - [f(-1) + f(1)]$$

$$E_2 = \int_{-1}^{1} f(x)dx - \frac{1}{2}\left(f(-1) + 2f(0) + f(1)\right)$$

and $|x|$ is the absolute value of $x \in \mathbb{R}$, then

1. $|E_1| < |E_2|$
2. $|E_1| = 2|E_2|$
3. $|E_1| = 4|E_2|$
4. $|E_1| = 8|E_2|$

43. Let $y(x)$ be a continuous solution of the initial value problem
$y' + 2y = f(x)$, $y(0) = 0$,
where $f(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & x > 1 \end{cases}$.
Then $y\left(\frac{3}{2}\right)$ is equal to

1. $\frac{\sinh(1)}{e^3}$
2. $\frac{\cosh(1)}{e^3}$
3. $\frac{\sinh(1)}{e^2}$
4. $\frac{\cosh(1)}{e^2}$

44. Let $a, b \in \mathbb{R}$ be such that $a^2 + b^2 \neq 0$. Then the Cauchy problem
$a\frac{\partial u}{\partial x} + b\frac{\partial u}{\partial y} = 1;\ x, y \in \mathbb{R}$
$u(x, y) = x$ on $ax + by = 1$

1. has more than one solution if either a or b is zero
2. has no solution
3. has a unique solution
4. has infinitely many solutions

45. The singular integral of the ODE $(xy' - y)^2 = x^2(x^2 - y^2)$ is

1. $y = x \sin x$
2. $y = x \sin(x + \frac{\pi}{4})$
3. $y = x$
4. $y = x + \frac{\pi}{4}$

46. The initial value problem
$y' = 2\sqrt{y},\ y(0) = a,$ has
1. a unique solution if $a < 0$
2. no solution if $a > 0$
3. infinitely many solutions if $a = 0$
4. a unique solution if $a \geq 0$

47. Consider two weightless, inextensible rols AB and BC, suspended at A and joined by a flexible joint at B. Then the degrees of freedom of the system is
1. 3
2. 4
3. 5
4. 6

48. Consider the initial value problem
$\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0, \quad u(0,y) = 4e^{-2y}.$
Then the value of $u(1,1)$ is
1. $4e^{-2}$
2. $4e^{2}$
3. $2e^{-4}$
4. $4e^{4}$

UNIT - 4

49. Suppose X_n, X are random variables such that X_n converges in distribution to X and $(-1)^n X_n$ also converges in distribution to X. Then
1. X must have a symmetric distribution.
2. X must be 0.
3. X must have a density.
4. X^2 must be a constant.

50. Ten balls are put in 6 slots at random. Then the expected total number of balls in the two extreme slots is
1. 10/6.
2. 10/3.
3. 1/6.
4. 6/10.

51. Assume that $X\sim$ Binomial (n,p) for some $n \geq 1$ and $0 < p < 1$ and $Y\sim$ Poisson (λ) for some $\lambda > 0$. Suppose $E[X] = E[Y]$. Then
1. $Var(X) = Var(Y)$
2. $Var(X) < Var(Y)$
3. $Var(Y) < Var(X)$
4. $Var(X)$ may be larger or smaller than $Var(Y)$ depending on the values of n, p and λ.

52. Suppose $X_i \mid \theta_i \sim N(\theta_i, \sigma^2), i = 1,2$ are independently distributed. Under the prior distribution, θ_1 and θ_2 are i.i.d $N(\mu, \tau^2)$, where σ^2, μ and τ^2 are known. Then which of the following is true about the marginal distributions of X_1 and X_2?
1. X_1 and X_2 are i.i.d $N(\mu, \tau^2 + \sigma^2)$.
2. X_1 and X_2 are not normally distributed.
3. X_1 and X_2 are $N(\mu, \tau^2 + \sigma^2)$ but they are not independent.
4. X_1 and X_2 are normally distributed but are not identically distributed.

53. $\{N(t): t \geq 0\}$ is a Poisson process with rate $\lambda > 0$. Let $X_n = N(n), n = 0,1,2,\cdots$ Which of the following is correct?
1. $\{X_n\}$ is a transient Markov chain.
2. $\{X_n\}$ is a recurrent Markov chain, but has no stationary distribution.
3. $\{X_n\}$ has a stationary distribution.
4. $\{X_n\}$ is an irreducible Markov chain.

54. Let $X_1, X_2, \cdots, X_n$ be independent and identically distributed random variables having an exponential distribution with mean $\frac{1}{\lambda}$.
Let $S_n = X_1 + X_2 + \cdots + X_n$ and $N = \inf\{n \geq 1: S_n > 1\}$. Then $Var(N)$ equals
1. 1.
2. λ.
3. λ^2.
4. ∞.

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55. Suppose there are k strata of $N = kM$ units each with size M. Draw a sample of size n_i with replacement from the i^{th} stratum and denote by $\bar{y}_i$ the sample mean of the study variable selected in the i^{th} stratum, $i = 1,2,\cdots,k$. Define

$\bar{y}_s = \frac{1}{k}\sum_{i=1}^{k}\bar{y}_i$ and $\bar{y}_w = \frac{\sum_{i=1}^{k} n_i\bar{y}_i}{n}$.

Which of the following is necessarily true?

1. $\bar{y}_s$ is unbiased but $\bar{y}_w$ is not unbiased for the population mean.
2. $\bar{y}_s$ is not unbiased but $\bar{y}_w$ is unbiased for the population mean.
3. Both $\bar{y}_s$ and $\bar{y}_w$ are unbiased for the population mean.
4. Neither $\bar{y}_s$ nor $\bar{y}_w$ is unbiased for the population mean.

56. Let X, Y be independent random variables and let $Z = \frac{X-Y}{2} + 3$. If X has characteristic function φ and Y has characteristic function ψ, then Z has characteristic function θ where

1. $\theta(t) = e^{-i3t}\varphi(2t)\psi(-2t)$.
2. $\theta(t) = e^{i3t}\varphi\left(\frac{t}{2}\right)\psi\left(-\frac{t}{2}\right)$.
3. $\theta(t) = e^{-i3t}\varphi\left(\frac{t}{2}\right)\psi\left(\frac{t}{2}\right)$.
4. $\theta(t) = e^{-i3t}\varphi\left(\frac{t}{2}\right)\psi\left(\frac{-t}{2}\right)$.

57. Consider the model $Y_i = i\beta + \varepsilon_i, i = 1,2,3$ where $\varepsilon_1, \varepsilon_2, \varepsilon_3$ are independent with mean 0 and variance $\sigma^2, 2\sigma^2, 3\sigma^2$ respectively. Which of the following is the best linear unbiased estimate of β?

1. $\frac{y_1+2y_2+3y_3}{6}$.
2. $\frac{6}{11}\left(y_1 + \frac{y_2}{2} + \frac{y_3}{3}\right)$.
3. $\frac{y_1+y_2+y_3}{6}$.
4. $\frac{3y_1+2y_2+y_3}{10}$.

58. Consider a Balanced Incomplete Block Design (BIBD) with parameters (b, k, v, r, λ). Which of the following cannot possibly be the parameters of a BIBD?

1. $(b-1, k-\lambda, b-k, k, \lambda)$.
2. $(b, v-k, v, b-r, b-2r+\lambda)$.
3. $\left(\frac{v(v-1)}{2}, 2, v, v-1, 1\right)$.
4. $(k, b, r, v, \lambda - 1)$.

59. Let $X_1, X_2, \cdots, X_7$ be a random sample from $N(\mu, \sigma^2)$ where μ and σ^2 are unknown. Consider the problem of testing $H_0: \mu = 2$ against $H_1: \mu > 2$. Suppose the observed values of $x_1, x_2, \cdots, x_7$ are 1.2, 1.3, 1.7, 1.8, 2.1, 2.3, 2.7. If we use the Uniformly Most Powerful test, which of the following is true?

1. H_0 is accepted both at 5% and 1% levels of significance.
2. H_0 is rejected both at 5% and 1% levels of significance.
3. H_0 is rejected at 5% level of significance, but accepted at 1% level of significance.
4. H_0 is rejected at 1% level of significance, but accepted at 5% level of significance.

60. Let $Y = (Y_1, \cdots, Y_n)'$ have the multivariate normal distribution $N_n(0, I)$. Which of the following is the covariance matrix of the conditional distribution of Y given

$$\sum_{i=1}^{n} Y_i?$$

(**1** denotes the $n \times 1$ vector with all elements 1.)

1. I.
2. $I + \frac{\mathbf{1}\mathbf{1}'}{n}$.
3. $I - \frac{\mathbf{1}\mathbf{1}'}{n}$.
4. $\frac{\mathbf{1}\mathbf{1}'}{n}$.

PART 'C'

UNIT - 1

61. Let $A = \begin{pmatrix} a & b & c \\ 0 & a & d \\ 0 & 0 & a \end{pmatrix}$ be a 3×3 matrix where a, b, c, d are integers. Then, we must have:
1. If $a \neq 0$, there is a polynomial $p \in \mathbb{Q}[x]$ such that $p(A)$ is the inverse of A.
2. For each polynomial $q \in \mathbb{Z}[x]$, the matrix $q(A) = \begin{pmatrix} q(a) & q(b) & q(c) \\ 0 & q(a) & q(d) \\ 0 & 0 & q(a) \end{pmatrix}$.
3. If $A^n = 0$ for some positive integer n, then $A^3 = 0$.
4. A commutes with every matrix of the form $\begin{pmatrix} a' & 0 & c' \\ 0 & a' & 0 \\ 0 & 0 & a' \end{pmatrix}$.

62. Let f be a bounded function on $\mathbb{R}$ and $a \in \mathbb{R}$. For $\delta > 0$, let $\omega(a, \delta) = \sup|f(x) - f(a)|$, $x \in [a - \delta, a + \delta]$. Then
1. $\omega(a, \delta_1) \leq \omega(a, \delta_2)$ if $\delta_1 \leq \delta_2$.
2. $\lim_{\delta \to 0+} \omega(a, \delta) = 0$ for all $a \in \mathbb{R}$.
3. $\lim_{\delta \to 0+} \omega(a, \delta)$ need not exist.
4. $\lim_{\delta \to 0+} \omega(a, \delta) = 0$ if and only if f is continuous at a.

63. Let S be the set of 3×3 real matrices A with $A^T A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. Then the set S contains
1. a nilpotent matrix.
2. a matrix of rank one.
3. a matrix of rank two.
4. a non-zero skew-symmetric matrix.

64. Consider non-zero vector spaces V_1, V_2, V_3, V_4 and linear transformations $\phi_1: V_1 \to V_2$, $\phi_2: V_2 \to V_3$, $\phi_3: V_3 \to V_4$ such that Ker $(\phi_1) = \{0\}$, Range $(\phi_1) =$ Ker (ϕ_2), Range $(\phi_2) =$ Ker (ϕ_3), Range $(\phi_3) = V_4$. Then
1. $\sum_{i=1}^{4} (-1)^i \dim V_i = 0$
2. $\sum_{i=2}^{4} (-1)^i \dim V_i > 0$.
3. $\sum_{i=1}^{4} (-1)^i \dim V_i < 0$.
4. $\sum_{i=1}^{4} (-1)^i \dim V_i \neq 0$.

65. For $n \geq 1$, let $g_n(x) = \sin^2\left(x + \frac{1}{n}\right)$, $x \in [0, \infty)$ and $f_n(x) = \int_0^x g_n(t)dt$. Then
1. $\{f_n\}$ converges pointwise to a function f on $[0, \infty)$, but does not converge uniformly on $[0, \infty)$.
2. $\{f_n\}$ does not converge pointwise to any function on $[0, \infty)$.
3. $\{f_n\}$ converges uniformly on $[0,1]$.
4. $\{f_n\}$ converges uniformly on $[0, \infty)$.

66. Let $S: \mathbb{R}^n \to \mathbb{R}^n$ be given by $S(v) = \alpha v$ for a fixed $\alpha \in \mathbb{R}$, $\alpha \neq 0$. Let $T: \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation such that $B = \{v_1, \ldots, v_n\}$ is a set of linearly independent eigenvectors of T. Then
1. The matrix of T with respect to B is diagonal.
2. The matrix of $(T - S)$ with respect to B is diagonal.

3. The matrix of T with respect to B is not necessarily diagonal, but is upper *triangular*.
4. The matrix of T with respect to B is diagonal but the matrix of $(T - S)$ with respect to B is not diagonal.

67. Let $F: \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ be the function $F(x,y) = \langle Ax, y\rangle$, where $\langle , \rangle$ is the standard inner product of $\mathbb{R}^n$ and A is a $n \times n$ real matrix. Here D denotes the total derivative. Which of the following statements are correct?
1. $(DF(x,y))(u,v) = \langle Au, y\rangle + \langle Ax, v\rangle$.
2. $(DF(x,y))(0,0) = 0$.
3. $DF(x,y)$ may not exist for some $(x,y) \epsilon \mathbb{R}^n \times \mathbb{R}^n$.
4. $DF(x,y)$ does not exist at $(x,y) = (0,0)$.

68. Which of the following are subspaces of the vector space $\mathbb{R}^3$?
1. $\{(x,y,z): x + y = 0\}$.
2. $\{(x,y,z): x - y = 0\}$.
3. $\{(x,y,z): x + y = 1\}$.
4. $\{(x,y,z): x - y = 1\}$.

69. An $n \times n$ complex matrix A satisfies $A^k = I_n$, the $n \times n$ identity matrix, where k is a positive integer > 1. Suppose 1 is not an eigenvalue of A. Then which of the following statements are necessarily true?
1. A is diagonalizable.
2. $A + A^2 + \cdots + A^{k-1} = 0$, the $n \times n$ zero matrix.
3. $tr(A) + tr(A^2) + \cdots + tr(A^{k-1}) = -n$.
4. $A^{-1} + A^{-2} + \cdots + A^{-(k-1)} = -I_n$.

70. Let $\underline{u}$ be a real $n \times 1$ vector satisfying $\underline{u}'\underline{u} = 1$, where $\underline{u}'$ is the transpose of $\underline{u}$. Define $A = I - 2\,\underline{u}\,\underline{u}'$ where I is the n^{th} order identity matrix. Which of the following statements are true?
1. A is singular.
2. $A^2 = A$.
3. Trace $(A) = n - 2$.
4. $A^2 = I$.

71. Let a be a positive real number. Which of the following integrals are convergent?
1. $\int_0^a \frac{1}{x^4} dx$.
2. $\int_0^a \frac{1}{\sqrt{x}} dx$.
3. $\int_4^\infty \frac{1}{x \log_e x} dx$.
4. $\int_5^\infty \frac{1}{x(\log_e x)^2} dx$.

72. Let A be an invertible 4×4 real matrix. Which of the following are NOT true?
1. Rank $A = 4$.
2. For every vector $b \epsilon \mathbb{R}^4$, $Ax = b$ has exactly one solution.
3. dim (nullspace A) ≥ 1.
4. 0 is an eigenvalue of A.

73. Which of the following sets of functions are uncountable? ($\mathbb{N}$ stands for the set of natural numbers.)
1. $\{f | f: \mathbb{N} \to \{1,2\}\}$.
2. $\{f | f: \{1,2\} \to \mathbb{N}\}$.
3. $\{f | f: \{1,2\} \to \mathbb{N}, f(1) \leq f(2)\}$.
4. $\{f | f: \mathbb{N} \to \{1,2\}, f(1) \leq f(2)\}$

74. For $n \geq 2$, let $a_n = \frac{1}{n \log n}$. Then
1. The sequence $\{a_n\}_{n=2}^\infty$ is convergent.
2. The series $\sum_{n=2}^\infty a_n$ is convergent.
3. The series $\sum_{n=2}^\infty a_n^2$ is convergent.
4. The series $\sum_{n=2}^\infty (-1)^n a_n$ is convergent.

75. Which of the following sets in $\mathbb{R}^2$ have positive Lebesgue measure?
$\left[\text{For two sets } A, B \subseteq \mathbb{R}^2, A + B = \{a + b \mid a \epsilon A, b \epsilon B\}\right]$

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1. $S = \{(x,y) \mid x^2 + y^2 = 1\}$
2. $S = \{(x,y) \mid x^2 + y^2 < 1\}$
3. $S = \{(x,y) | x = y\} + \{(x,y) \mid x = -y\}$
4. $S = \{(x,y) | x = y\} + \{(x,y) \mid x = y\}$

76. Let $p_n(x) = x^n$ for $x \in \mathbb{R}$ and let $\wp = span\{p_0, p_1, p_2, \ldots\}$. Then
1. $\wp$ is the vector space of all real valued continuous functions on $\mathbb{R}$.
2. $\wp$ is a subspace of all real valued continuous functions on $\mathbb{R}$.
3. $\{p_0, p_1, p_2, \ldots\}$ is a linearly independent set in the vector space of all continuous functions on $\mathbb{R}$.
4. Trigonometric functions belong to $\wp$.

77. Let $f: \mathbb{R}^n \to \mathbb{R}^n$ be a continuous function such that $\int_{\mathbb{R}^n} |f(x)| dx < \infty$.
Let A be a real $n \times n$ invertible matrix and for $x, y \in \mathbb{R}^n$, let $\langle x, y\rangle$ denote the standard inner product in $\mathbb{R}^n$. Then
$\int_{\mathbb{R}^n} f(Ax) e^{i\langle y,x\rangle} dx =$
1. $\int_{\mathbb{R}^n} f(x) e^{i\langle (A^{-1})^T y, x\rangle} \frac{dx}{|det\, A|}$
2. $\int_{\mathbb{R}^n} f(x) e^{i\langle A^T y, x\rangle} \frac{dx}{|\det A|}$.
3. $\int_{\mathbb{R}^n} f(x) e^{i\langle (A^T)^{-1} y, x\rangle} dx$.
4. $\int_{\mathbb{R}^n} f(x) e^{i\langle A^{-1} y, x\rangle} \frac{dx}{|\det A|}$.

78. Let $\{a_0, a_1, a_2, \ldots\}$ be a sequence of real numbers.
For any $k \geq 1$, let $s_n = \sum_{k=0}^{n} a_{2k}$. Which of the following statements are correct?
1. If $\lim_{n\to\infty} s_n$ exists, then $\sum_{m=0}^{\infty} a_m$ exists.
2. If $\lim_{n\to\infty} s_n$ exists, then $\sum_{m=0}^{\infty} a_m$ need not exist.
3. If $\sum_{m=0}^{\infty} a_m$ exists, then $\lim_{n\to\infty} s_n$ exists.
4. If $\sum_{m=0}^{\infty} a_m$ exists, then $\lim_{n\to\infty} s_n$ need not exist.

UNIT - 2

79. Let f be an analytic function defined on the open unit disc in $\mathbb{C}$. Then f is constant if
1. $f\left(\frac{1}{n}\right) = 0$ for all $n \geq 1$.
2. $f(z) = 0$ for all $|z| = \frac{1}{2}$.
3. $f\left(\frac{1}{n^2}\right) = 0$ for all $n \geq 1$.
4. $f(z) = 0$ for all $z \in (-1, 1)$.

80. Let f be an entire function. Which of the following statements are correct?
1. f is constant if the range of f is contained in a straight line.
2. f is constant if f has uncountably many zeros.
3. f is constant if f is bounded on $\{z \in \mathbb{C}: \text{Re}(z) \leq 0\}$
4. f is constant if the real part of f is bounded.

81. Let $C([0,1])$ be the ring of all real valued continuous functions on $[0,1]$. Which of the following statements are true?
1. $C([0,1])$ is an integral domain.
2. The set of all functions vanishing at 0 is a maximal ideal.
3. The set of all functions vanishing at both 0 and 1 is a prime ideal.
4. If $f \in C([0,1])$ is such that $(f(x))^n = 0$ for all $x \in [0,1]$ for some $n > 1$, then $f(x) = 0$ for all $x \in [0,1]$.

82. Let p be a polynomial in 1-complex variable. Suppose all zeroes of p are in the upper half plane $H = \{z\epsilon\mathbb{C} \mid Im\,(z) > 0\}$. Then

1. $Im \frac{p'(z)}{p(z)} > 0$ for $z\epsilon\mathbb{R}$.
2. $Re\, i \frac{p'(z)}{p(z)} < 0$ for $z\epsilon\mathbb{R}$.
3. $Im \frac{p'(z)}{p(z)} > 0$ for $z\epsilon\mathbb{C}$, with $Im\, z < 0$.
4. $Im \frac{p'(z)}{p(z)} > 0$ for $z\epsilon\mathbb{C}$, with $Im\, z > 0$.

83. Which of the following primes satisfy the congruence
$a^{24} \equiv 6a + 2 \bmod 13$?

1. 41
2. 47
3. 67
4. 83

84. Which of the following polynomials are irreducible in the ring $\mathbb{Z}[x]$ of polynomials in one variable with integer coefficients?

1. $x^2 - 5$.
2. $1 + (x+1) + (x+1)^2 + (x+1)^3 + (x+1)^4$.
3. $1 + x + x^2 + x^3 + x^4$.
4. $1 + x + x^2 + x^3$.

85. Consider the set $\mathbb{Z}$ of integers, with the topology τ in which a subset is closed if and only if it is empty, or $\mathbb{Z}$, or finite. Which of the following statements are true?

1. τ is the subspace topology induced from the usual topology on $\mathbb{R}$.
2. $\mathbb{Z}$ is compact in the topology τ.
3. $\mathbb{Z}$ is Hausdorff in the topology τ.
4. Every infinite subset of $\mathbb{Z}$ is dense in the topology τ.

86. Let $\sigma: \{1,2,3,4,5\} \to \{1,2,3,4,5\}$ be a permutation (one-to-one and onto function) such that
$\sigma^{-1}(j) \le \sigma(j) \quad \forall j, 1 \le j \le 5$.

Then which of the following are true?

1. $\sigma \circ \sigma(j) = j$ for all j, $1 \le j \le 5$.
2. $\sigma^{-1}(j) = \sigma(j)$ for all j, $1 \le j \le 5$.
3. The set $\{k: \sigma(k) \ne k\}$ has an even number of elements.
4. The set $\{k: \sigma(k) = k\}$ has an odd number of elements.

87. Determine which of the following polynomials are irreducible over the indicated rings.

1. $x^5 - 3x^4 + 2x^3 - 5x + 8$ over $\mathbb{R}$.
2. $x^3 + 2x^2 + x + 1$ over $\mathbb{Q}$.
3. $x^3 + 3x^2 - 6x + 3$ over $\mathbb{Z}$.
4. $x^4 + x^2 + 1$ over $\mathbb{Z}/2\mathbb{Z}$.

88. If x, y and z are elements of a group such that $xyz = 1$, then

1. $yzx = 1$.
2. $yxz = 1$.
3. $zxy = 1$.
4. $zyx = 1$.

89. Which of the following cannot be the class equation of a group of order 10?

1. $1 + 1 + 1 + 2 + 5 = 10$.
2. $1 + 2 + 3 + 4 = 10$.
3. $1 + 2 + 2 + 5 = 10$.
4. $1 + 1 + 2 + 2 + 2 + 2 = 10$.

90. Consider the following subsets of the complex plane:

$$\Omega_1 = \left\{ C\epsilon\mathbb{C} : \begin{bmatrix} 1 & C \\ \bar{C} & 1 \end{bmatrix} \text{ is non-negative definite (or equivalently positive semi-definite)} \right\}$$

$$\Omega_2 = \left\{ C\epsilon\mathbb{C} : \begin{bmatrix} 1 & C & C \\ \bar{C} & 1 & C \\ \bar{C} & \bar{C} & 1 \end{bmatrix} \right.$$

is non negative definite

(or equivalently positive semi – definite)

Let $\bar{D} = \{z \in \mathbb{C} \mid |z| \leq 1\}$. Then

1. $\Omega_1 = \bar{D}, \Omega_2 = \bar{D}$.
2. $\Omega_1 \neq \bar{D}, \Omega_2 = \bar{D}$.
3. $\Omega_1 = \bar{D}, \Omega_2 \neq \bar{D}$.
4. $\Omega_1 \neq \bar{D}\ \Omega_2 \neq \bar{D}$.

UNIT - 3

91. Let P be a continuous function on $\mathbb{R}$ and W the Wronskian of two linearly independent solutions y_1 and y_2 of the ODE:
$\frac{d^2y}{dx^2} + (1+x^2)\frac{dy}{dx} + P(x)y = 0, x \in \mathbb{R}$.
Let $W(1) = a$, $W(2) = b$ and $W(3) = c$, then

1. $a < 0$ and $b > 0$
2. $a < b < c$ or $a > b > c$
3. $\frac{a}{|a|} = \frac{b}{|b|} = \frac{c}{|c|}$
4. $0 < a < b$ and $b > c > 0$

92. The critical point of the system $\frac{dx}{dt} = -4x - y$, $\frac{dy}{dt} = x - 2y$ is an

1. asymptotically stable node
2. unstable node
3. asymptotically stable spiral
4. unstable spiral

93. The extremal of the functional $\int_0^{\alpha}(y'^2 - y^2)dx$ that passes through $(0,0)$ and $(\alpha, 0)$ has a

1. weak minimum if $\alpha < \pi$
2. strong minimum if $\alpha < \pi$
3. weak minimum if $\alpha > \pi$
4. strong minimum if $\alpha > \pi$

94. For the initial value problem
$\frac{dy}{dx} = y^2 + \cos^2 x$, $x > 0$
$y(0) = 0$,
The largest interval of existence of the solution predicted by Picard's theorem is:

1. [0, 1]
2. [0, 1/2]
3. [0, 1/3]
4. [0, 1/4]

95. Which of the following are complete integrals of the partial differential equation $pqx + yq^2 = 1$?

1. $z = \frac{x}{a} + \frac{ay}{x} + b$
2. $z = \frac{x}{b} + \frac{ay}{x} + b$
3. $z^2 = 4(ax + y) + b$
4. $(z - b)^2 = 4(ax + y)$

96. The function

$$G(x,\zeta) = \begin{cases} a + b\log\zeta, & 0 < x \leq \zeta \\ c + d\log x, & \zeta \leq x \leq 1 \end{cases}$$

is a Green's function for $xy'' + y' = 0$, subject to y being bounded as $x \to 0$ and $y(1) = y'(1)$, if

1. $a = 1,\ b = 1,\ c = 1,\ d = 1$
2. $a = 1,\ b = 0,\ c = 1,\ d = 0$
3. $a = 0,\ b = 1,\ c = 0,\ d = 1$
4. $a = 0,\ b = 0,\ c = 0,\ d = 0$

97. The second order partial differential equation $u_{xx} + x\,u_{yy} = 0$ is

1. elliptic for $x > 0$
2. hyperbolic for $x > 0$
3. elliptic for $x < 0$
4. hyperbolic for $x < 0$

98. For an arbitrary continuously differentiable function f, which of the following is a general solution of $z(px - qy) = y^2 - x^2$

1. $x^2 + y^2 + z^2 = f(xy)$
2. $(x + y)^2 + z^2 = f(xy)$
3. $x^2 + y^2 + z^2 = f(y - x)$
4. $x^2 + y^2 + z^2 = f((x + y)^2 + z^2)$

99. A particle of mass m is constrained to move on the surface of a cylinder $x^2 + y^2 = a^2$ under the influence of a force directed towards the origin and proportional to the distance of the particle from the origin. Then

1. the angular momentum about z-axis is constant
2. the angular momentum about z-axis is not constant
3. the motion is simple harmonic in z-direction
4. the motion is not simple harmonic in z-direction

100. The extremal of the functional $I = \int_0^{x_1} y^2(y')^2 dx$ that passes through (0,0) and (x_1, y_1) is

1. a constant function
2. a linear function of x
3. part of a parabola
4. part of an ellipse

101. The following numerical integration formula is exact for all polynomials of degree less than or equal to 3

1. Trapezoidal rule
2. Simpson's $\frac{1}{3}rd$ rule
3. Simpson's $\frac{3}{8}th$ rule
4. Gauss-Legendre 2 point formula

102. For the integral equation $y(x) = 1 + x^3 + \int_0^x K(x,t)y(t)dt$ with kernel $K(x,t) = 2^{x-t}$, the iterated kernel $K_3(x,t)$ is

1. $2^{x-t}(x - t)^2$
2. $2^{x-t}(x - t)^3$
3. $2^{x-t-1}(x - t)^2$
4. $2^{x-t-1}(x - t)^3$

UNIT - 4

103. Let X and Y be random variables with joint cumulative distribution function $F(x,y)$. Then which of the following conditions are sufficient for $(x,y) \epsilon \mathbb{R}^2$ to be a point of continuity of F?

1. $P(X = x, Y = y) = 0$.
2. Either $P(X = x) = 0$ or $P(Y = y) = 0$.
3. $P(X = x) = 0$ and $P(Y = y) = 0$.
4. $P(X = x, Y \leq y) = 0$ and $P(X \leq x, Y = y) = 0$.

104. Suppose X has density $f(x|\theta) = \frac{1}{\theta}e^{-x/\theta}, x > 0$ where $\theta > 0$ is unknown. Define Y as follows: $Y = k$ if $k \leq X < k + 1$, $k = 0, 1, 2, \cdots$ Then the distribution of Y is

1. normal.
2. binomial.
3. Poisson.
4. geometric.

105. Suppose X has density $f(x|\theta)$ where θ is 0 or 1. Also, $f(x|0) = 1$ if $0 < x < 1$, and 0 otherwise, $f(x|1) = \frac{1}{2\sqrt{x}}$ if $0 < x < 1$ and 0 otherwise.

To test $H_0: \theta = 0$ versus $H_1: \theta = 1$ at level $\alpha, 0 < \alpha < 1$, the Most Powerful test

1. rejects H_0 if $X > 1 - \alpha$.
2. rejects H_0 if $X < \alpha$.
3. rejects H_0 if $X < \sqrt{\alpha}$.
4. has power $\sqrt{\alpha}$.

106. Let (X, Y) have the joint discrete distribution such that $X \mid Y = y \sim$ Binomial $(y, 0.5)$ and $Y \sim \text{Poisson}(\lambda), \lambda > 0$, where λ is an unknown parameter. Let $T = T(X, Y)$ be any unbiased estimator of λ. Then

1. $Var(T) \leq Var(Y)$ for all λ.
2. $Var(T) \geq Var(Y)$ for all λ.
3. $Var(T) \geq \lambda$ for all λ.
4. $Var(T) = Var(Y)$ for all λ.

107. Consider a sample of size one, say X, from a population with pdf

$$f_\theta(x) = \frac{2}{\theta^2}(x - \theta) \qquad \theta \leq x \leq 2\theta, \theta > 0$$
$$= 0 \qquad \text{otherwise}$$

Which of the following is/are confidence interval(s) for θ with confidence coefficient $1 - \alpha$?

1. $\left[\frac{X}{2}, \frac{X}{1+\sqrt{\alpha}}\right]$.
2. $\left[\frac{X}{1+\sqrt{1-\frac{\alpha}{2}}}, \frac{X}{1+\sqrt{\frac{\alpha}{2}}}\right]$.
3. $\left[\frac{X}{1+\sqrt{1-\alpha}}, X\right]$.
4. $\left[\frac{X}{1+\sqrt{1-\frac{\alpha}{4}}}, \frac{X}{1+\sqrt{\frac{3\alpha}{4}}}\right]$.

108. Consider a Markov Chain with state space $S = \{0, 1, 2, 3\}$ and with transition probability matrix P given by

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 2/3 & 0 & 1/3 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{pmatrix} \end{array}$$

Then

1. 1 is a recurrent state.
2. 0 is a recurrent state.
3. 3 is a recurrent state.
4. 2 is a recurrent state.

109. Let X and Y be independent normal random variables with mean 0 and variance 1. Let the characteristic function of XY be denoted by φ. Then

1. $\varphi(2) = 1/2$.
2. φ is an even function.
3. $\varphi(t)\varphi\left(\frac{1}{t}\right) = |t|$ for all $t \neq 0$.
4. $\varphi(t) = E\left(e^{-t^2Y^2/2}\right)$.

110. Let X_1 and X_2 be independent and identically distributed normal random variables with mean 0 and variance 1. Let U_1 and U_2 be independent and identically distributed $U(0,1)$ random variables, independent of X_1, X_2. Define $Z = \frac{X_1U_1 + X_2U_2}{\sqrt{U_1^2 + U_2^2}}$. Then,

1. $E(Z) = 0$.
2. $Var(Z) = 1$.
3. Z is standard Cauchy.
4. $Z \sim N(0,1)$

111. Consider the pdf

$$f(x; \theta, \sigma) = \frac{0.9}{\sigma}\varphi\left(\frac{x-\theta}{\sigma}\right) + 0.1\,\varphi(x - \theta),$$

where $-\infty < \theta < \infty$ and $\sigma > 0$ are unknown parameters and φ denotes the pdf of $N(0,1)$. Let $X_1, X_2, \cdots, X_n$ be a random sample from this probability distribution. Then which of the following is (are) correct?

1. This model is not parametric.
2. Method of moments estimators for θ and σ exist.
3. An unbiased estimator of θ exists.
4. Consistent estimators of θ do not exist.

112. Suppose $X_1, X_2, \cdots$ are independent random variables. Assume that $X_1, X_3, \cdots$ are identically distributed with mean μ_1 and variance $\sigma_1{}^2$, while $X_2, X_4, \cdots$ are identically distributed with mean μ_2 variance σ_2^2. Let $S_n = X_1 + X_2 + \cdots + X_n$. Then $\frac{S_n - a_n}{b_n}$ converges in distribution to $N(0,1)$ if

1. $a_n = \frac{n(\mu_1+\mu_2)}{2}$ and $b_n = \sqrt{n}\sqrt{\frac{\sigma_1^2+\sigma_2^2}{2}}$.
2. $a_n = \frac{n(\mu_1+\mu_2)}{2}$ and $b_n = \frac{n(\sigma_1+\sigma_2)}{2}$.
3. $a_n = n(\mu_1 + \mu_2)$ and $b_n = \sqrt{n}\frac{(\sigma_1+\sigma_2)}{2}$.
4. $a_n = n(\mu_1 + \mu_2)$ and $b_n = \sqrt{n}\sqrt{\frac{\sigma_1^2+\sigma_2^2}{2}}$.

113. Consider the linear model $Y \sim N_n(X\beta, \sigma^2 I)$, where X is a $n \times (k+1)$ matrix of rank $k+1 < n$. Let $\hat{\beta}$ and $\hat{\sigma}^2$ be the maximum likelihood estimators of β and σ^2 respectively. Then which of the following statements are true?

1. $cov(\hat{\beta}) = \sigma^2 X'X$
2. $\hat{\beta}$ and $\hat{\sigma}^2$ are independently distributed
3. $\hat{\sigma}^2$ is sufficient for σ^2
4. $\hat{\sigma}^2 = Y'AY$ where A is a suitable matrix of rank $(n-k-1)$.

114. Let Y_1, Y_2, Y_3, Y_4 be i.i.d standard normal variables. Which of the following has Wishart distribution with 2 d.f.?

1. $\begin{bmatrix} Y_1^2 + Y_2^2 & Y_2^2 + Y_3^2 \\ Y_2^2 + Y_3^2 & Y_3^2 + Y_4^2 \end{bmatrix}$.
2. $\begin{bmatrix} Y_1^2 & Y_2^2 \\ Y_3^2 & Y_4^2 \end{bmatrix}$.
3. $\begin{bmatrix} Y_1^2 + Y_2^2 & 0 \\ 0 & Y_3^2 + Y_4^2 \end{bmatrix}$.
4. $\begin{bmatrix} Y_1^2 + Y_2^2 & Y_1Y_3 + Y_2Y_4 \\ Y_1Y_3 + Y_2Y_4 & Y_3^2 + Y_4^2 \end{bmatrix}$.

115. To check whether a premium version of petrol gives better fuel efficiency, a random sample of 10 cars of a single model were tested with both premium and standard petrol. Let the mileages obtained be denoted by $(X_1, Y_1), \cdots, (X_{10}, Y_{10})$, where X_i denotes the mileage from standard and Y_i from the premium for the i^{th} car. We want to test
H_0: There is no difference in fuel efficiency between the two versions of petrol

Versus

H_1: Premium petrol gives better fuel efficiency

Let $D_i = Y_i - X_i$, $\bar{D} = \bar{Y} - \bar{X}$, $S_i =$ Rank of D_i when $|D_i|$ are ordered. It is felt that fuel efficiency measurements are not normally distributed and hence a nonparametric test is to be proposed. Then which of the following can be considered suitable statistics for this purpose?

1. $\bar{Y} - \bar{X}$.
2. Numbers of positive $D_i's$.
3. Sum of S_i corresponding to positive D_i.
4. $\frac{\bar{D}}{\sqrt{\Sigma(D_i - \bar{D})^2}}$.

116. Let $\Sigma = ((\sigma_{ij}))$ be an $n \times n$ symmetric and positive definite matrix such that $\sigma_{ij} \neq 0$ for all i, j. Which of the following matrices will always be the covariance matrix of a multivariate normal random vector?

1. Σ.
2. The matrix with the (ij) th element σ_{ij}^2 for each i, j.
3. The matrix with (ij)th element $\frac{1}{\sigma_{ij}}$ for each i, j.
4. Σ^{-1}.

117. Let $X(t)$ = number of customers in the system at time t in an $M/M/C$ queueing model, with $C = 3$, arrival rate $\lambda > 0$ and service rate $\mu > 0$. Which of the following is/are true?

1. $\{X(t)\}$ is a birth and death process with constant birth and death rates.
2. If $\{X(t)\}$ has a stationary distribution, then $\lambda < 3\mu$.
3. If $\lambda < 3\mu$, then the stationary distribution is a geometric distribution with parameter $\frac{\lambda}{3\mu}$.
4. The number of customers undergoing service at time t is min $\{X(t), 3\}$.

118. Consider the linear model

$Y_1 = \mu_1 - \mu_2 + \epsilon_1$
$Y_2 = \mu_2 - \mu_3 + \epsilon_2$
$\vdots$
$Y_{n-1} = \mu_{n-1} - \mu_n + \epsilon_{n-1}$
$Y_n = \mu_n - \mu_1 + \epsilon_n$

where $\mu_1, \dots, \mu_n$ are unknown parameters and $\epsilon_1, \dots, \epsilon_n$ are uncorrelated with mean 0 and common variance. Let Y be the column vector $(Y_1, Y_2, \cdots, Y_n)'$ and $\bar{Y} = \frac{1}{n}\sum_{i=1}^n Y_i$. Which of the following are correct?

1. If $E(c'Y) = 0$, then all elements of c' are equal.
2. The best linear unbiased estimator of $\mu_1 - \mu_3$ is $Y_1 + Y_2$.
3. The best linear unbiased estimator of $\mu_2 - \mu_3$ is $Y_2 - \bar{Y}$.
4. All linear functions $d_1\mu_1 + \cdots + d_n\mu_n$ are estimable.

119. Consider a random sample $(X_1, Y_1), \dots, (X_n, Y_n)$ from the bivariate normal distribution with $E(X_i) = \mu = E(Y_i)$, $Var(X_i) = \sigma^2 = Var(Y_i)$ and $Cov(X_i, Y_i) = \rho\sigma^2$ for all i. Let $\hat{\mu}, \hat{\sigma}^2$ and $\hat{\rho}$ denote the maximum likelikhood estimators of μ, σ^2 and ρ respectively. Also,

$S_X^2 = \sum_{i=1}^n (X_i - \bar{X})^2$, $S_Y^2 = \sum_{i=1}^n (Y_i - \bar{Y})^2$ and

$S_{XY} = \sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})$.

Then

1. $X_1 - Y_1$ and $X_1 + Y_1$ are independent.
2. $\hat{\mu} = \frac{1}{2}(\bar{X} + \bar{Y}), \hat{\sigma}^2 = \frac{1}{2n}(S_X^2 + S_Y^2), \hat{\rho} = \frac{2S_{XY}}{S_X^2 + S_Y^2}$.
3. $\hat{\sigma}^2(1 + \hat{\rho}) = \frac{1}{2n}(S_X^2 + S_Y^2 + 2S_{XY})$.
4. $\hat{\gamma}^2(1 - \hat{\rho}) = \frac{1}{2n}(S_X^2 + S_Y^2 - 2S_{XY})$.

120. Let Y_1, Y_2, Y_3 be uncorrelated observations with common variance σ^2 and expectations given by $E(Y_1) = \theta_0 + \theta_1$, $E(Y_2) = \theta_0 + \theta_2$, $E(Y_3) = \theta_0 + \theta_3$, where $\theta_i's$ are unknown parameters. In the framework of the linear model which of the following statement(s) is (are) true?

1. Each of $\theta_0, \theta_1, \theta_2$ and θ_3 is individually estimable.
2. $\sum_{i=0}^3 \theta_i$ is estimable.
3. $\theta_1 - \theta_2$, $\theta_1 - \theta_3$ and $\theta_2 - \theta_3$ are each estimable.
4. The error sum of squares is zero.

Qn. Set C	Key
1	1
2	2
3	3
4	4
5	4
6	2
7	4
8	1
9	3
10	2
11	4
12	4
13	4
14	2
15	4
16	2
17	1
18	1
19	1
20	2
21	1
22	3
23	3
24	2
25	3
26	2
27	3
28	1
29	4
30	1
31	1
32	4
33	4
34	3
35	2
36	2
37	2
38	3
39	3
40	3
41	4

Qn. Set C	Key
42	3
43	3
44	3
45	3
46	3
47	2
48	2
49	1
50	2
51	2
52	1
53	1
54	2
55	1
56	2
57	3
58	4
59	1
60	3
61	1,3,4
62	1,4
63	1,2
64	1,2
65	3,4
66	1,2
67	1,2
68	1,2
69	1,3,4
70	3,4
71	2,4
72	3,4
73	1,4
74	1,3,4
75	2,3
76	2,3
77	1
78	2,4
79	1,2,3,4
80	1,2,4
81	2,4
82	1,2,3

Qn. Set C	Key
83	1,3
84	1,2,3
85	2,4
86	1,2,3,4
87	2,3
88	1,3
89	1,2,4
90	3
91	2,3
92	1
93	1,2
94	2
95	1,4
96	1
97	1,4
98	1,2,4
99	1,3
100	3
101	2,3,4
102	3
103	3,4
104	4
105	2,4
106	2,3
107	1,2,3,4
108	2,4
109	2,4
110	1,2,4
111	2,3
112	1
113	2,4
114	4
115	2,3
116	1,2,4
117	2,4
118	1,3
119	1,2,3,4
120	3,4