PART 'A'

1. The houses of three sisters lie in the same row, but the middle sister does not live in the middle house. In the morning, the shadow of the eldest sister's house falls on the youngest sister's house. What can be concluded for sure?
   1. The youngest sister lives in the middle.
   2. The eldest sister lives in the middle.
   3. Either the youngest or the eldest sister lives in the middle.
   4. The youngest sister's house lies on the east of the middle sister's house.

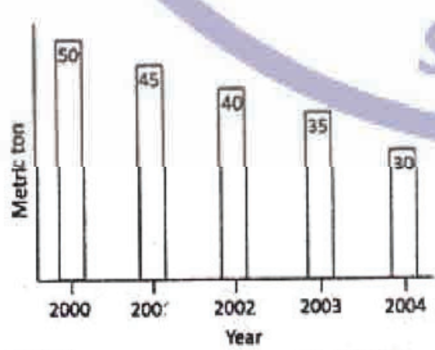
2. A woman starts shopping with Rs. X and Y paise, spends Rs. 3.50 and is left with Rs. $2Y$ and $2X$ paise. The amount she started with is

1. Rs. 48.24	2. Rs. 26.64
3. Rs. 32.14	4. Rs. 23.42

3. A mine supplies 10000 tons of copper ore, containing an average of 1.5 wt% copper, to a smelter every day. The smelter extracts 80% of the copper from the ore on the same day. What is the production of copper in tons/day?

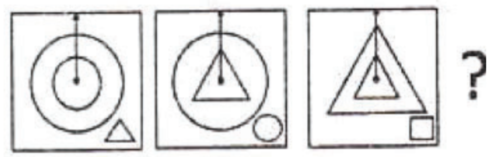
1. 80	2. 12
3. 120	4. 150

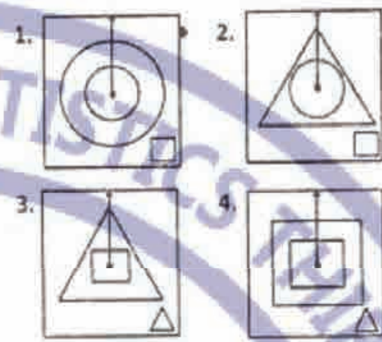
4. Wheat production of a country over a number of years is shown. Which year recorded highest percent reduction in production over the previous year?



1. 2001	2. 2002
3. 2003	4. 2004

5. What is the next pattern in the given sequence?



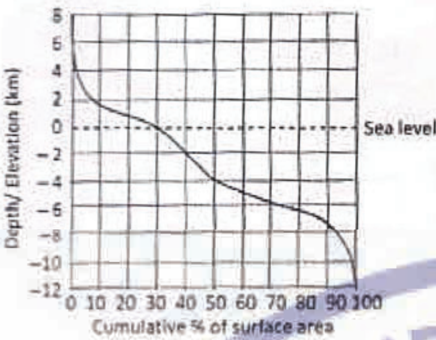


6. A person completely under sea water tracks the Sun. Compared to an observer above water, which of the following observations would be made by the underwater observer?
   1. Neither the time of sunrise or sunset nor the angular span of the horizon changes.
   2. Sunrise is delayed, sunset is advanced, but there is no change in the angular span of the horizon.
   3. Sunrise and sunset times remain unchanged, but the angular span of the horizon shrinks.
   4. The duration of the day and the angular span of the horizon, both decrease.

7. A man sells three articles A, B, C and gains 10% on A, 20% on B and loses 10% on C. He breaks even when combined selling prices of A and C are considered, whereas he gains 5% when combined selling prices of B and C are considered. What is his net loss or gain on the sale of all the articles?

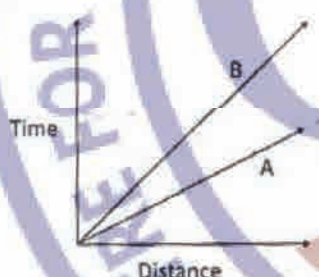
1. 10% gain	2. 20% gain
3. 10.66% gain	4. 6.66% gain

8. Based on the distribution of surface area of the Earth at different elevations and depths (with reference to sea-level) shown in the figure, which of the following is FALSE?

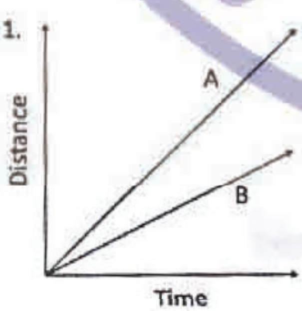


1. Larger proportion of the surface of the Earth is below sea-level
2. Of the surface area above sea-level, larger proportion lies below 2 km elevation
3. Of the surface area below sea-level, smaller proportion lies below 4 km depth
4. Distance from sea level to the maximum depth is greater than that to the maximum elevation

9. Time-distance graph of two objects A and B are shown.



If the axes are interchanged, then the same information is shown by

1.


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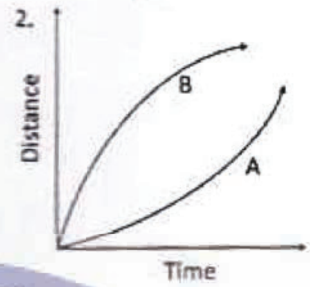

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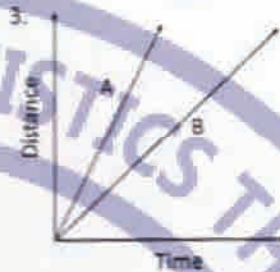

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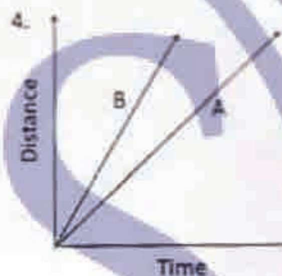


10. A chocolate salesman is travelling with 3 boxes with 30 chocolates in each box. During his journey he encounters 30 toll booths. Each toll booth inspector takes one chocolate per box that contains chocolate(s), as tax. What is the largest number of chocolates he can be left with after passing through all toll booths?

1. 0	2. 30
3. 25	4. 20

11. A milkman adds 10 litres of water to 90 litres of milk. After selling $1/5^{th}$ of the total quantity, he adds water equal to the quantity he has sold. The proportion of water to milk he sells now would be

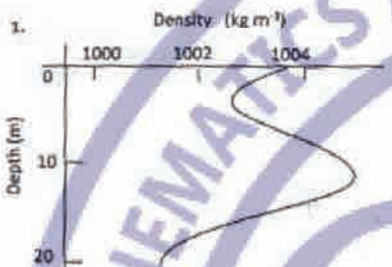
1. 72:28	2. 28:72
3. 20:80	4. 30:70

12. Two coconuts have spherical space inside their kernels, with the first having an inner diameter twice that of the other. The larger one is half filled with liquid, while the smaller is completely filled. Which of the following statements is correct?

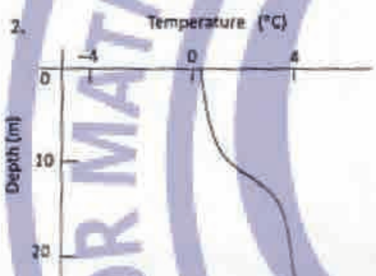
1. The larger coconut contains 4 times the liquid in the smaller one.
2. The larger coconut contains twice the liquid in the smaller one.
3. The coconuts contain equal volumes of liquid.
4. The smaller coconut contains twice the liquid in the larger one.

13. Which of the following graphs represents a stable fresh water lake?(i.e., no vertical motion of water)

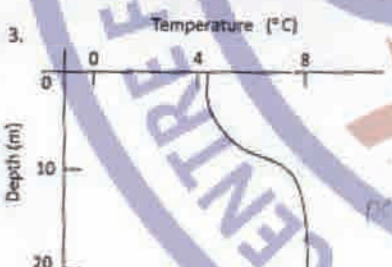
1.



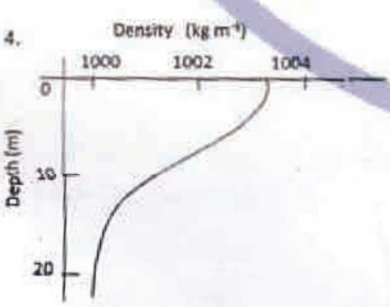
2.



3.



4.



14. A tiger usually stalks its prey from a direction that is upwind of the prey. The reason for this is

1. the wind aids its final burst for killing the prey
2. the wind carries the scent of the prey to the tiger and helps the tiger locate the prey easily
3. the upwind area usually has denser vegetation and better camouflage
4. the upwind location aids the tiger by not letting its smell reach the prey

15. A cellphone tower radiates 1W power while the handset transmitter radiates 0.1 mW power. The correct comparison of the radiation energy received by your head from a tower 100m away (E_1) and that from a handset held to your ear (E_2) is

1. $E_1 >> E_2$
2. $E_2 >> E_1$
3. $E_1 = E_2$ for communication to be established
4. insufficient data even for a rough comparison

16. The pitch of a spring is 5 mm. The diameter of the spring is 1 cm. The spring spins about its axis with a speed of 2 rotations/s. The spring appears to be moving parallel to its axis with a speed of

1. 1 mm/s
2. 5 mm/s
3. 6 mm/s
4. 10 mm/s

17. The dimensions of a floor are 18 × 24. What is the smallest number of identical square tiles that will pave the entire floor without the need to break any tile?

1. 6
2. 24
3. 8
4. 12

18. To determine the number of parrots in a sparse population, an ecologist captures 30 parrots and puts rings around their necks and releases them. After a week he captures 40 parrots and finds that 8 of them have rings on their necks. What approximately is the parrot population?

1. 70
2. 150
3. 160
4. 100

19. The mid-point of the arc of a semicircle is connected by two straight lines to the ends of the diameter as shown. What is the ratio of the shaded area to the area of the triangle?

1. $\frac{\pi}{4}-1$
2. $\frac{\pi-1}{4}$
3. $\pi - 1/2$
4. $2\pi - 1/4$

20. Why is there low fish population in lakes that have large hyacinth growth?
1. Hyacinth prevents sunlight from reaching the depths of the lake.
2. Decaying matter from hyacinth consumes dissolved oxygen in copious amounts.
3. Hyacinth is not a suitable food for fishes.
4. Hyacinth releases toxins in the water.

PART 'B'

Unit-1

21. Consider the sets of sequences
$X = \{(x_n): x_n \epsilon \{0,1\}, n\epsilon \mathbb{N}\}$ and
$Y = \{(x_n)\epsilon X: x_n = 1$ for at most finitely many $n\}$.
Then
1. X is countable, Y is finite.
2. X is uncountable, Y is countable.
3. X is countable, Y is countable.
4. X is uncountable, Y is uncountable.

22. The matrix
$$\begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$$ is
1. positive definite.
2. non-negative definite but not positive definite.
3. negative definite.
4. neither negative definite nor positive definite.

23. Let $f: \mathbb{R}^2 \to \mathbb{R}^2$ be given by $f(x,y) = (x^2, y^2 + \sin x)$. Then the derivative of f at (x,y) is the linear transformation given by
1. $\begin{pmatrix} 2x & 0 \\ \cos x & 2y \end{pmatrix}$
2. $\begin{pmatrix} 2x & 0 \\ 2y & \cos x \end{pmatrix}$
3. $\begin{pmatrix} 2y & \cos x \\ 2x & 0 \end{pmatrix}$
4. $\begin{pmatrix} 2x & 2y \\ 0 & \cos x \end{pmatrix}$

24. A function $f: \mathbb{R}^2 \to \mathbb{R}$ is defined by $f(x,y) = xy$. Let $v = (1,2)$ and $a = (a_1, a_2)$ be two elements of $\mathbb{R}^2$. The directional derivative of f in the direction of v at a is:
1. $a_1 + 2a_2$
2. $a_2 + 2a_1$
3. $\frac{a_2}{2} + a_1$
4. $\frac{a_1}{2} + a_2$

25.
$$\lim_{n\to\infty} \frac{1}{n^4} \sum_{j=0}^{2n-1} j^3 \text{ equals}$$
1. 4
2. 16
3. 1
4. 8

26. $f: \mathbb{R} \to \mathbb{R}$ is such that $f(0) = 0$ and $\left|\frac{df}{dx}(x)\right| \le 5$ for all x. We can conclude that $f(1)$ is in
1. $(5,6)$.
2. $[-5,5]$.
3. $(-\infty,-5) \cup (5,\infty)$.
4. $[-4,4]$.

27. Which of the following subsets of $\mathbb{R}^4$ is a basis of $\mathbb{R}^4$?
$B_1 = \{(1,0,0,0),(1,1,0,0),(1,1,1,0),(1,1,1,1)\}$
$B_2 = \{(1,0,0,0),(1,2,0,0),(1,2,3,0),(1,2,3,4)\}$
$B_3 = \{(1,2,0,0),(0,0,1,1),(2,1,0,0),(-5,5,0,0)\}$
1. B_1 and B_2 but not B_3
2. B_1, B_2 and B_3
3. B_1 and B_3 but not B_2
4. Only B_1

28. Let

$$D_1 = det\begin{pmatrix} a & b & c \\ x & y & z \\ p & q & r \end{pmatrix} \text{ and}$$

$$D_2 = det\begin{pmatrix} -x & a & -p \\ y & -b & q \\ z & -c & r \end{pmatrix}. \text{ Then}$$

1. $D_1 = D_2$
2. $D_1 = 2D_2$
3. $D_1 = -D_2$
4. $2D_1 = D_2$

29. Consider the matrix
$A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, where $\theta = \frac{2\pi}{31}$

Then A^{2015} equals

1. A
2. I
3. $\begin{pmatrix} \cos 13\theta & \sin 13\theta \\ -\sin 13\theta & \cos 13\theta \end{pmatrix}$
4. $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

30. Let J denote the matrix of order $n \times n$ with all entries 1 and let B be a $(3n) \times (3n)$ matrix given by $B = \begin{pmatrix} 0 & 0 & J \\ 0 & J & 0 \\ J & 0 & 0 \end{pmatrix}$

Then the rank of B is

1. $2n$
2. $3n - 1$
3. 2
4. 3

31. Which of the following sets of functions from $\mathbb{R}$ to $\mathbb{R}$ is a vector space over $\mathbb{R}$?

$S_1 = \{f \mid \lim_{x\to 3} f(x) = 0\}$

$S_2 = \left\{g \middle| \lim_{x\to 3} g(x) = 1\right\}$

$S_3 = \left\{h \middle| \lim_{x\to 3} h(x) exists\right\}$

1. Only S_1
2. Only S_2
3. S_1 and S_3 but not S_2
4. All the three are vector spaces

32. Let A be an $n \times m$ matrix with each entry equal to $+1, -1$ or 0 such that every column has exactly one $+1$ and exactly one -1. We can conclude that

1. Rank $A \le n - 1$
2. Rank $A = m$
3. $n \le m$
4. $n - 1 \le m$

Unit-2

33. The radius of convergence of the series

$$\sum_{n=1}^{\infty} z^{n^2}$$

is

1. 0
2. ∞
3. 1
4. 2

34. Let C be the circle $|z| = 3/2$ in the complex plane that is oriented in the counter clockwise direction. The value of a for which

$$\int_C \left(\frac{z+1}{z^2 - 3z + 2} + \frac{a}{z-1}\right) dz = 0$$

is

1. 1
2. -1
3. 2
4. -2

35. Suppose f and g are entire functions and $g(z) \neq 0$ for all $z \in \mathbb{C}$. If $|f(z)| \le |g(z)|$, then we conclude that

1. $f(z) \neq 0$ for all $z \in \mathbb{C}$.
2. f is a constant function.
3. $f(0) = 0$.
4. for some $C \in \mathbb{C}$, $f(z) = Cg(z)$.

36. Let f be a holomorphic function on $0 < |z| < \epsilon$, $\epsilon > 0$ given by a convergent Laurent series

$$\sum_{n=-\infty}^{\infty} a_n z^n.$$

Given also that

$\lim_{z\to 0} |f(z)| = \infty$,

We can conclude that

1. $a_{-1} \neq 0$ and $a_{-n} = 0$ for all $n \ge 2$
2. $a_{-N} \neq 0$ for some $N \ge 1$ and $a_{-n} = 0$ for all $n > N$

3. $a_{-n} = 0$ for all $n \geq 1$
4. $a_{-n} \neq 0$ for all $n \geq 1$

37. Given a natural number $n > 1$ such that $(n-1)! \equiv -1 \ (mod\ n)$. We can conclude that
1. $n = p^k$ where p is prime, $k > 1$.
2. $n = pq$ where p and q are distinct primes.
3. $n = pqr$ where p, q, r are distinct primes.
4. $n = p$ where p is a prime.

38. Let S_n denote the permutation group on n symbols and A_n be the subgroup of even permutations. Which of the following is true?
1. There exists a finite group which is not a subgroup of S_n for any $n \geq 1$.
2. Every finite group is a subgroup of A_n for some $n \geq 1$.
3. Every finite group is a quotient of A_n for some $n \geq 1$.
4. No finite abelian group is a quotient of S_n for $n > 3$.

39. What is the number of non-singular 3×3 matrices over $\mathbb{F}_2$, the finite field with two elements?

1. 168.   2. 384.
3. 2^3.   4. 3^2.

40. Let G be an open set in $\mathbb{R}^n$. Two points $x, y \in G$ are said to be equivalent if they can be joined by a continuous path completely lying inside G. Number of equivalence classes is
1. only one.
2. at most finite.
3. at most countable.
4. can be finite, countable or uncountable.

Unit-3

41. Let $(x(t), y(t))$ satisfy the system of ODEs

$$\frac{dx}{dt} = -x + ty$$

$$\frac{dy}{dt} = tx - y$$

If $(x_1(t), y_1(t))$ and $(x_2(t), y_2(t))$ are two solutions and

$$\Phi(t) = x_1(t)y_2(t) - x_2(t)y_1(t)$$

then $\frac{d\Phi}{dt}$ is equal to

1. -2Φ.   2. 2Φ.
3. $-\Phi$.   4. Φ.

42. The boundary value problem $x^2y'' - 2xy' + 2y = 0$, subject to the boundary conditions $y(1) + \alpha y'(1) = 1$, $y(2) + \beta y'(2) = 2$ has a unique solution if
1. $\alpha = -1,\ \beta = 2$.
2. $\alpha = -1,\ \beta = -2$.
3. $\alpha = -2,\ \beta = 2$.
4. $\alpha = -3,\ \beta = \frac{2}{3}$.

43. The PDE $x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial y^2} = 0$ is
1. hyperbolic for $x > 0,\ y < 0$.
2. elliptic for $x > 0,\ y < 0$.
3. hyperbolic for $x > 0,\ y > 0$.
4. elliptic for $x < 0,\ y > 0$.

44. Let $u(x,t)$ satisfy the initial boundary value problem

$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2};\quad x \in (0,1),\quad t > 0$

$u(x,0) = \sin(\pi x);\quad x \in [0,1]$

$u(0,t) = u(1,t) = 0,\ t > 0$

Then for $x \in (0,1)$, $u\left(x, \frac{1}{\pi^2}\right)$ is equal to

1. $e\ \sin(\pi x)$.   2. $e^{-1}\sin(\pi x)$.
3. $\sin(\pi x)$.   4. $\sin(\pi^{-1}x)$.

45. The values of α and β, such that

$$x_{n+1} = \alpha x_n\left(3 - \frac{x_n^2}{a}\right) + \beta x_n\left(1 + \frac{a}{x_n^2}\right)$$

has 3rd order convergence to $\sqrt{a}$, are
1. $\alpha = \frac{3}{8},\ \beta = \frac{1}{8}$.
2. $\alpha = \frac{1}{8},\ \beta = \frac{3}{8}$.
3. $\alpha = \frac{2}{8},\ \beta = \frac{2}{8}$.
4. $\alpha = \frac{1}{4},\ \beta = \frac{3}{4}$.

46. If

$$J[y] = \int_1^2 (y'^2 + 2yy' + y^2)dx, \ y(1) = 1$$

and $y(2)$ is arbitrary then the extremal is

1. e^{x-1}.
2. e^{x+1}.
3. e^{1-x}.
4. e^{-x-1}.

47. Let ϕ satisfy

$$\phi(x) = f(x) + \int_0^x \sin(x-t)\phi(t)\,dt.$$

Then ϕ is given by

1. $\phi(x) = f(x) + \int_0^x (x-t)f(t)dt$
2. $\phi(x) = f(x) - \int_0^x (x-t)f(t)dt$
3. $\phi(x) = f(x) - \int_0^x \cos(x-t)f(t)dt$
4. $\phi(x) = f(x) - \int_0^x \sin(x-t)f(t)dt$

48. A bead slides without friction on a frictionless wire in the shape of a cycloid with equation $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$, $0 \le \theta \le 2\pi$.
Then the Lagrangian function is

1. $ma^2(1+\cos\theta)\dot{\theta}^2 - mga(1+\cos\theta)$
2. $ma^2(1-\cos\theta)\dot{\theta}^2 - mga(1+\cos\theta)$
3. $ma^2(1-\cos\theta)\dot{\theta}^2 + mga(1+\cos\theta)$
4. $ma^2(1+\cos\theta)\dot{\theta}^2 - mga(1-\cos\theta)$

Unit-4

49. There are two boxes. Box 1 contains 2 red balls and 4 green balls. Box 2 contains 4 red balls and 2 green balls. A box is selected at random and a ball is chosen randomly from the selected box. If the ball turns out to be red, what is the probability that Box 1 had been selected?

1. $\frac{1}{2}$
2. $\frac{1}{3}$
3. $\frac{2}{3}$
4. $\frac{1}{6}$

50. For any two events A and B, which of the following relations always holds?

1. $P^2(A \cap B^C) + P^2(A \cap B) + P^2(A^C) \ge \frac{1}{3}$
2. $P^2(A \cap B^C) + P^2(A \cap B) + P^2(A^C) = \frac{1}{3}$
3. $P^2(A \cap B^C) + P^2(A \cap B) + P^2(A^C) = 1$
4. $P^2(A \cap B^C) + P^2(A \cap B) + P^2(A^C) \le \frac{1}{3}$

51. Suppose customers arrive in a shop according to a Poisson process with rate 4 per hour. The shop opens at 10:00 am. If it is given that the second customer arrives at 10:40 am, what is the probability that no customer arrived before 10:30 am?

1. $\frac{1}{4}$
2. e^{-2}
3. $\frac{1}{2}$
4. $e^{-1/2}$

52. Suppose $X_1, X_2, \cdots, X_n$ is a random sample from a distribution with probability density function $f(x) = 3x^2 I_{(0,1)}(x)$, where $I_{(0,1)}(z) = \begin{cases} 1 & if\ z \in (0,1) \\ 0 & otherwise \end{cases}$.
What is the probability density function $g(y)$ of $Y = \min\{X_1, X_2, \cdots, X_n\}$?

1. $g(y) = 3ny^{3n-1}\, I_{(0,1)}(y)$.
2. $g(y) = 1 - (1-y^3)^n\, I_{(0,1)}(y)$.
3. $g(y) = (1-y^3)^n\, I_{(0,1)}(y)$.
4. $g(y) = 3ny^2(1-y^3)^{n-1}\, I_{(0,1)}(y)$.

53. $X_1, X_2, \cdots, X_n$ are independent and identically distributed $N(\theta, 1)$ random variables, where θ takes only integer values i.e. $\theta \in \{\cdots, -2, -1, 0, 1, 2, \cdots\}$. Which of the following is the maximum likelihood estimator of θ?

1. $\bar{X}$
2. Integer closest to $\bar{X}$
3. Integer part of $\bar{X}$, (Largest integer $\le \bar{X}$)
4. median of $(X_1, X_2, \cdots, X_n)$

54. For a random variable X, with $E(X) > 0$, the coefficient of variation ρ is defined as $\rho = \frac{\sigma_X}{E(X)}$ where σ_x^2 is the variance of X. Suppose $X_1, X_2, \cdots, X_n$ are independent samples from a normal population with mean 2 and unknown coefficient of variation ρ. It is desired to test $H_0: \rho \le 5$ against $H_1: \rho > 5$. The likelihood ratio test is of the form: Reject H_0 if

1. $\Sigma(X_i - 2)^2 > C$.
2. $\Sigma(X_i - 2)^2 < C$.
3. $\frac{\Sigma(X_i - \bar{X})^2}{\bar{X} - 2} > C$.
4. $\frac{\Sigma(X_i - \bar{X})^2}{\bar{X} - 2} < C$.

55. $(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)$ are data on X-cultivable land in a district and Y-the area actually under cultivation, both measured in square feet. Let $\hat{\alpha}, \hat{\beta}$ be the least squares estimates of α, β in the model $Y = \alpha + \beta x + \varepsilon$, where ε is the random error. If the data are converted to square meters, then

1. $\hat{\alpha}$ may change but $\hat{\beta}$ will not.
2. $\hat{\beta}$ may change but $\hat{\alpha}$ will not.
3. both $\hat{\alpha}$ and $\hat{\beta}$ may change.
4. Neither $\hat{\alpha}$ nor $\hat{\beta}$ will change.

56. Suppose in a one-way analysis of variance model, the sum of squares of all the group means is 0 (Assume that all the observations are not same). Then the value of the usual F-test statistic for testing the equality of means

1. cannot be determined from the above information.
2. is undefined.
3. is 0.
4. is 1.

57. Let $(X_1, X_2, \cdots, X_p)$ be a random vector with mean $\underset{\sim}{\mu}$ and a positive definite dispersion matrix Σ. Then the coefficient vector $(l_1, l_2, \cdots, l_p)$ of the first principal component

$$\sum_{i=1}^{p} l_i X_i \text{ is}$$

1. the vector of all the eigenvalues of Σ.
2. the eigenvector corresponding to the smallest eigenvalue of Σ.
3. the eigenvector corresponding to the largest eigenvalue of Σ.
4. the vector of all the eigenvalues of Σ^{-1}.

58. A simple random sample (without replacement) of size n is drawn from a finite population of size $N(\ge 7)$. What is the probability that the 4^{th} population unit is included in the sample but the 6^{th} population unit is not included in the sample?

1. $\frac{n(n-1)}{N(N-1)}$
2. $\frac{n(N-n)}{N(N-1)}$
3. $\frac{(n-1)(N-n+1)}{N(N-1)}$
4. $\frac{n}{N}$

59. (v, b, r, k, λ) are the standard parameters of a balanced incomplete block design $(BIBD)$. Which of the following (v, b, r, k, λ) can be parameters of a $BIBD$?

1. $(v, b, r, k, \lambda) = (44, 33, 9, 12, 3)$
2. $(v, b, r, k, \lambda) = (17, 45, 8, 3, 1)$
3. $(v, b, r, k, \lambda) = (35, 35, 17, 17, 9)$
4. $(v, b, r, k, \lambda) = (16, 24, 9, 6, 3)$

60. Consider an M/M/1 Queue with arrival rate λ and service rate μ with $\mu > \lambda$. What is the probability that no customer exited the system before time 5?

1. $\frac{\mu e^{-5\lambda} - \lambda e^{-5\mu}}{\mu - \lambda}$
2. $e^{-5\lambda} - e^{-5\mu}$
3. $e^{-5\lambda} + (1 - e^{-5\lambda})\frac{e^{-5\mu}}{5\mu}$
4. $e^{-5\mu} + (1 - e^{-5\mu})\frac{e^{-5\lambda}}{5\lambda}$

PART 'C'

Unit-1

61. Let A be a $n \times n$ non-singular matrix with real entries. Let $B = A^T$ denote the traspose of A. Which of the following matrices are positive definite?
1. $A + B$
2. $A^{-1} + B^{-1}$
3. AB
4. ABA

62. Let $s \in (0,1)$. Then decide which of the following are true.
1. $\forall m \in \mathbb{N}, \exists n \in \mathbb{N} s.t. \; s > m/n$
2. $\forall m \in \mathbb{N}, \exists n \in \mathbb{N} s.t. \; s < m/n$
3. $\forall m \in \mathbb{N}, \exists n \in \mathbb{N} s.t. \; s = m/n$
4. $\forall m \in \mathbb{N}, \exists n \in \mathbb{N} s.t. \; s = m + n$

63. Let $f_n(x) = (-x)^n$, $x \in [0,1]$. Then decide which of the following are true.
1. there exists a pointwise convergent subsequence of f_n.
2. f_n has no pointwise convergent subsequence.
3. f_n converges pointwise everywhere.
4. f_n has exactly one pointwise convergent subsequence.

64. Which of the following are true for the function $f(x) = \sin(x)\sin\left(\frac{1}{x}\right), x \in (0,1)$?
1. $\underline{\lim}_{x\to 0} f(x) = \overline{\lim}_{x\to 0} f(x)$
2. $\underline{\lim}_{x\to 0} f(x) < \overline{\lim}_{x\to 0} f(x)$
3. $\underline{\lim}_{x\to 0} f(x) = 1$
4. $\overline{\lim}_{x\to 0} f(x) = 0$

65. Find out which of the following series converge uniformly for $x \in (-\pi, \pi)$.
1. $\sum_{n=1}^{\infty} \frac{e^{-n|x|}}{n^3}$
2. $\sum_{n=1}^{\infty} \frac{\sin(xn)}{n^5}$
3. $\sum_{n=1}^{\infty} \left(\frac{x}{n}\right)^n$
4. $\sum_{n=1}^{\infty} \frac{1}{((x+\pi)n)^2}$

66. Decide which of the following functions are uniformly continuous on $(0,1)$.
1. $f(x) = e^x$
2. $f(x) = x$
3. $f(x) = \tan\left(\frac{\pi x}{2}\right)$
4. $f(x) = \sin(x)$

67. Let $\chi_A(x)$ denote the function which is 1 if $x \in A$ and 0 otherwise. Consider

$$f(x) = \sum_{n=1}^{200} \frac{1}{n^6} \chi_{\left[0, \frac{n}{200}\right]}(x), \quad x \in [0,1].$$

Then $f(x)$ is
1. Riemann integrable on $[0,1]$.
2. Lebesgue integrable on $[0,1]$.
3. is a continuous function on $[0,1]$.
4. is a monotone function on $[0,1]$.

68. A function $f(x,y)$ on $\mathbb{R}^2$ has the following partial derivatives

$$\frac{\partial f}{\partial x}(x,y) = x^2, \quad \frac{\partial f}{\partial y}(x,y) = y^2.$$

Then
1. f has directional derivatives in all directions everywhere.
2. f has a derivative at all points.
3. f has directional derivative only along the direction $(1,1)$ everywhere.
4. f does not have directional derivatives in any direction everywhere.

69. Let d_1, d_2 be the following metrics on $\mathbb{R}^n$.

$$d_1(x,y) = \sum_{i=1}^{n} |x_i - y_i|, \quad d_2(x,y) = \left(\sum_{i=1}^{n} |x_i - y_i|^2\right)^{1/2}.$$

Then decide which of the following is a metric on $\mathbb{R}^n$.
1. $d(x,y) = \frac{d_1(x,y) + d_2(x,y)}{1 + d_1(x,y) + d_2(x,y)}$
2. $d(x,y) = d_1(x,y) - d_2(x,y)$
3. $d(x,y) = d_1(x,y) + d_2(x,y)$
4. $d(x,y) = e^{\pi} d_1(x,y) + e^{-\pi} d_2(x,y)$

70. Let A be the following subset of $\mathbb{R}^2$:
$A = \{(x,y) : (x+1)^2 + y^2 \le 1\} \cup \left\{(x,y) : y = x \sin\frac{1}{x}, x > 0\right\}$.
Then
1. A is connected
2. A is compact
3. A is path connected
4. A is bounded

71. Let

$$\ell^\infty = \{\underline{a} = (a_k)_{k\ge1} : a_k \in \mathbb{C}, \sup_k |a_k| \equiv \|\underline{a}\|_\infty < \infty\}$$

$$\ell^2 = \{\underline{a} = (a_k)_{k\ge1} : a_k \in \mathbb{C} : \left(\sum |a_k|^2\right)^{1/2} \equiv \|\underline{a}\|_2 < \infty\}$$

Define a map $T : \ell^\infty \to \ell^2$ as

$$T\,\underline{a} = \left\{a_1, \frac{a_2}{2}, \frac{a_3}{3}, \dots\right\}.$$

Which of the following statements is true?
1. T is a continuous linear map
2. T maps ℓ^∞ onto ℓ^2
3. T^{-1} exists and is continuous
4. T is uniformly continuous

72. Let $A = [a_{ij}]$ be an $n \times n$ matrix such that a_{ij} is an integer for all i, j. Let $AB = I$ with $B = [b_{ij}]$ (where I is the identity matrix). For a square matrix C, det C denotes its determinant. Which of the following statements is true?
1. If $\det A = 1$ then $\det B = 1$.
2. A sufficient condition for each b_{ij} to be an integer is that $\det A$ is an integer.
3. B is always an integer matrix.
4. A necessary condition for each b_{ij} to be an integer is $\det A \in \{-1, +1\}$.

73. Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ and let α_n and β_n denote the two eigenvalues of A^n such that $|\alpha_n| \ge |\beta_n|$. Then
1. $\alpha_n \to \infty$ as $n \to \infty$
2. $\beta_n \to 0$ as $n \to \infty$
3. β_n is positive if n is even.
4. β_n is negative if n is odd.

74. Let M_n denote the vector space of all $n \times n$ real matrices. Among the following subsets of M_n, decide which are linear subspaces.
1. $V_1 = \{A \in M_n : A \text{ is nonsingular}\}$
2. $V_2 = \{A \in M_n : \det(A) = 0\}$
3. $V_3 = \{A \in M_n : \text{trace}(A) = 0\}$
4. $V_4 = \{BA : A \in M_n\}$, where B is some fixed matrix in M_n.

75. If P and Q are invertible matrices such that $PQ = -QP$, then we can conclude that
1. $Tr(P) = Tr(Q) = 0$
2. $Tr(P) = Tr(Q) = 1$
3. $Tr(P) = -Tr(Q)$
4. $Tr(P) \ne Tr(Q)$

76. Let n be an odd number ≥ 7. Let $A = [a_{ij}]$ be an $n \times n$ matrix with $a_{i,i+1} = 1$ for all $i = 1, 2, \dots, n-1$ and $a_{n,1} = 1$. Let $a_{ij} = 0$ for all the other pairs (i,j). Then we can conclude that
1. A has 1 as an eigenvalue.
2. A has -1 as an eigenvalue.
3. A has at least one eigenvalue with multiplicity ≥ 2.
4. A has no real eigenvalues.

77. Let W_1, W_2, W_3 be three distinct subspaces of $\mathbb{R}^{10}$ such that each W_i has dimension 9. Let $W = W_1 \cap W_2 \cap W_3$. Then we can conclude that

1. W may not be a subspace of $\mathbb{R}^{10}$
2. $\dim W \leq 8$
3. $\dim W \geq 7$
4. $\dim W \leq 3$

78. Let A be a real symmetric matrix. Then we can conclude that
1. A does not have 0 as an eigenvalue
2. All eigenvalues of A are real
3. If A^{-1} exists, then A^{-1} is real and symmetric
4. A has at least one positive eigenvalue

Unit-2

79. Let $f(z)$ be the meromorphic function given by $\frac{z}{(1-e^z)\sin z}$. Then
1. $z = 0$ is a pole of order 2.
2. for every $k \in \mathbb{Z}, z = 2\pi ik$ is a simple pole.
3. for every $k \in \mathbb{Z}\setminus\{0\}, z = k\pi$ is a simple pole.
4. $z = \pi + 2\pi i$ is a pole.

80. Consider the polynomial

$$P(z) = \sum_{n=1}^{N} a_n z^n, \qquad 1 \leq N < \infty, \quad a_n \in \mathbb{R}\setminus\{0\}$$

Then, with $\mathbb{D} = \{w \in \mathbb{C}: |w| < 1\}$
1. $P(\mathbb{D}) \subseteq \mathbb{R}$
2. $P(\mathbb{D})$ is open
3. $P(\mathbb{D})$ is closed
4. $P(\mathbb{D})$ is bounded

81. Consider the polynomial

$$P(z) = \left(\sum_{n=0}^{5} a_n z^n\right)\left(\sum_{n=0}^{9} b_n z^n\right)$$

where $a_n, b_n \in \mathbb{R}\ \forall n, a_5 \neq 0, b_9 \neq 0$. Then counting roots with multiplicity we can conclude that $P(z)$ has
1. at least two real roots.
2. 14 complex roots.
3. no real roots.
4. 12 complex roots.

82. Let $\mathbb{D}$ be the open unit disc in $\mathbb{C}$. Let $g: \mathbb{D} \to \mathbb{D}$ be holomorphic, $g(0) = 0$, and

let $h(z) = \begin{cases} g(z)/z, & z \in \mathbb{D}, z \neq 0 \\ g'(0), & z = 0 \end{cases}$.

Which of the following statements are true?
1. h is holomorphic in $\mathbb{D}$.
2. $h(\mathbb{D}) \subseteq \overline{\mathbb{D}}$.
3. $|g'(0)| > 1$.
4. $|g(1/2)| \leq 1/2$.

83. Consider the following subsets of the group of 2×2 non-singular matrices over $\mathbb{R}$:

$$G = \left\{\begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in \mathbb{R},\ ad = 1\right\}$$

$$H = \left\{\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} : b \in \mathbb{R}\right\}.$$

Which of the following statements are correct?
1. G forms a group under matrix multiplication.
2. H is a normal subgroup of G.
3. The quotient group G/H is well-defined and is Abelian.
4. The quotient group G/H is well defined and is isomorphic to the group of 2×2 diagonal matrices (over $\mathbb{R}$) with determinant 1.

84. Let $\mathbb{C}$ be the field of complex numbers and $\mathbb{C}^*$ be the group of non zero complex numbers under multiplication. Then which of the following are true?
1. $\mathbb{C}^*$ is cyclic.
2. Every finite subgroup of $\mathbb{C}^*$ is cyclic.
3. $\mathbb{C}^*$ has finitely many finite subgroups.
4. Every proper subgroup of $\mathbb{C}^*$ is cyclic.

85. Let R be a finite non-zero commutative ring with unity. Then which of the following statements are necessarily true?
1. Any non-zero element of R is either a unit or a zero divisor.
2. There may exist a non-zero element of R which is neither a unit nor a zero divisor.
3. Every prime ideal of R is maximal.
4. If R has no zero divisors then order of any additive subgroup of R is a prime power.

86. Which of the following statements are true?

1. $\mathbb{Z}[x]$ is a principal ideal domain.
2. $\mathbb{Z}[x,y]/\langle y+1\rangle$ is a unique factorization domain.
3. If R is a principal ideal domain and $\mathfrak{p}$ is a non-zero prime ideal, then $R/\mathfrak{p}$ has finitely many prime ideals
4. If R is a principal ideal domain, then any subring of R containing 1 is again a principal ideal domain

87. Let R be a commutative ring with unity and $R[x]$ be the polynomial ring in one variable. For a non zero $f = \sum_{n=0}^{N} a_n x^n$, define $\omega(f)$ to be the smallest n such that $a_n \neq 0$. Also $\omega(0) = +\infty$. Then which of the following statements is/are true?

1. $\omega(f+g) \geq \min(\omega(f), \omega(g))$.
2. $\omega(fg) \geq \omega(f) + \omega(g)$.
3. $\omega(f+g) = \min(\omega(f), \omega(g))$, if $\omega(f) \neq \omega(g)$.
4. $\omega(fg) = \omega(f) + \omega(g)$, if R is an integral domain.

88. Let $\mathbb{F}_2$ be the finite field of order 2. Then which of the following statements are true?

1. $\mathbb{F}_2[x]$ has only finitely many irreducible elements.
2. $\mathbb{F}_2[x]$ has exactly one irreducible polynomial of degree 2.
3. $\mathbb{F}_2[x]/\langle x^2+1\rangle$ is a finite dimensional vector space over $\mathbb{F}_2$.
4. Any irreducible polynomial in $\mathbb{F}_2[x]$ of degree 5 has distinct roots in any algebraic closure of $\mathbb{F}_2$.

89. Let (X, d) be a metric space. Then

1. An arbitrary open set G in X is a countable union of closed sets.
2. An arbitrary open set G in X cannot be countable union of closed sets if X is connected.
3. An arbitrary open set G in X is a countable union of closed sets only if X is countable.
4. An arbitrary open set G in X is a countable union of closed sets only if X is locally compact.

90. Let $S = \{(x,y)\epsilon\mathbb{R}^2 \mid -1 \leq x \leq 1 \text{ and } -1 \leq y \leq 1\}$.
Let $T = S\setminus(0,0)$, the set obtained by removing the origin from S.
Let f be a continuous function from T to $\mathbb{R}$.
Choose all correct options.

1. Image of f must be connected.
2. Image of f must be compact.
3. Any such continuous function f can be extended to a continuous function from S to $\mathbb{R}$.
4. If f can be extended to a continuous function from S to $\mathbb{R}$ then the image of f is bounded.

Unit-3

91. Let $x: [0,3\pi] \to \mathbb{R}$ be a nonzero solution of the ODE
$x''(t) + e^{t^2}x(t) = 0$, for $t \in [0,3\pi]$.
Then the cardinality of the set $\{t \in [0,3\pi] : x(t) = 0\}$ is

1. equal to 1
2. greater than or equal to 2
3. equal to 2
4. greater than or equal to 3

92. Consider the initial value problem
$y'(t) = f(y(t)), \quad y(0) = a \in \mathbb{R}$ where $f: \mathbb{R} \to \mathbb{R}$.
Which of the following statements are necessarily true?

1. There exists a continuous function $f: \mathbb{R} \to \mathbb{R}$ and $a\epsilon\mathbb{R}$ such that the above problem does not have a solution in any neighbourhood of 0.
2. The problem has a unique solution for every $a\epsilon\mathbb{R}$ when f is Lipschitz continuous.
3. When f is twice continuously differentiable, the maximal interval of existence for the above initial value problem is $\mathbb{R}$.
4. The maximal interval of existence for the above problem is $\mathbb{R}$ when f is bounded and continuously differentiable.

93. Let $(x(t), y(t))$ satisfy for $t > 0$

$$\frac{dx}{dt} = -x + y, \quad \frac{dy}{dt} = -y, \quad x(0) = y(0) = 1.$$

Then $x(t)$ is equal to

1. $e^{-t} + t\ y(t)$
2. $y(t)$
3. $e^{-t}(1+t)$
4. $-y(t)$

94. Consider the wave equation for $u(x,t)$

$$\left.\begin{aligned} \frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0, (x,t) \in \mathbb{R} \times (0,\infty) \\ u(x,0) = f(x),\ x \in \mathbb{R} \\ \frac{\partial u}{\partial t}(x,0) = g(x),\ x \in \mathbb{R} \end{aligned}\right\}$$

Let u_i be the solution of the above problem with $f = f_i$ and $g = g_i$ for $i = 1,2$, where $f_i: \mathbb{R} \to \mathbb{R}$ and $g_i: \mathbb{R} \to \mathbb{R}$ are given C^2 functions satisfying $f_1(x) = f_2(x)$ and $g_1(x) = g_2(x)$, for every $x \in [-1,1]$. Which of the following statements are necessarily true?

1. $u_1(0,1) = u_2(0,1)$
2. $u_1(1,1) = u_2(1,1)$
3. $u_1\left(\frac{1}{2},\frac{1}{2}\right) = u_2\left(\frac{1}{2},\frac{1}{2}\right)$
4. $u_1(0,2) = u_2(0,2)$

95. Let $u: \mathbb{R}^2 \setminus \{(0,0)\} \to \mathbb{R}$ be a C^2 function satisfying $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, for all $(x,y) \neq (0,0)$. Suppose u is of the form $u(x,y) = f\left(\sqrt{x^2+y^2}\right)$, where $f: (0,\infty) \to \mathbb{R}$, is a nonconstant function, then

1. $\lim_{x^2+y^2 \to 0} |u(x,y)| = \infty$
2. $\lim_{x^2+y^2 \to 0} |u(x,y)| = 0$
3. $\lim_{x^2+y^2 \to \infty} |u(x,y)| = \infty$
4. $\lim_{x^2+y^2 \to \infty} |u(x,y)| = 0$

96. The Cauchy problem

$$\left.\begin{aligned} y\frac{\partial u}{\partial x} - x\frac{\partial u}{\partial y} = 0 \\ u = g \text{ on } \Gamma \end{aligned}\right\}$$

has a unique solution in a neighbourhood of Γ for every differentiable function $g: \Gamma \to \mathbb{R}$ if

1. $\Gamma = \{(x,0): x > 0\}$
2. $\Gamma = \{(x,y): x^2 + y^2 = 1\}$
3. $\Gamma = \{(x,y): x + y = 1, x > 1\}$
4. $\Gamma = \{(x,y): y = x^2,\ x > 0\}$

97. The order of linear multi step method $u_{j+1} = (1-a)u_j + au_{j-1} + \frac{h}{4}\{(a+3)u'_{j+1} + (3a+1)u'_{j-1}\}$ for solving $u' = f(x,u)$ is

1. 2 if $a = -1$
2. 2 if $a = -2$
3. 3 if $a = -1$
4. 3 if $a = -2$

98. The functional

$$J[y] = \int_0^1 (y'^2 + x^2)dx$$

where $y(0) = -1$ and $y(1) = 1$ on $y = 2x - 1$, has

1. weak minimum
2. weak maximum
3. strong minimum
4. strong maximum

99. Let $y(x)$ be a piecewise continuously differentiable function on $[0,4]$. Then the functional

$$J[y] = \int_0^4 (y' - 1)^2 (y' + 1)^2 dx$$

attains minimum if $y = y(x)$ is

1. $y = \frac{x}{2} \quad 0 \le x \le 4$
2. $y = \begin{cases} -x & 0 \le x \le 1 \\ x - 2 & 1 \le x \le 4 \end{cases}$
3. $y = \begin{cases} 2x, & 0 \le x \le 2 \\ -x + 6, & 2 \le x \le 4 \end{cases}$
4. $y = \begin{cases} x, & 0 \le x \le 3 \\ -x + 6 & 3 \le x \le 4 \end{cases}$

100. Which of the following are the characteristic numbers and the corresponding eigen-functions for the Fredholm homogeneous equation whose kernel is

$K(x,t) = \begin{cases}(x+1)t, & 0 \le x \le t, \\ (t+1)x, & t \le x \le 1\end{cases}$?

1. $1,\ e^x$
2. $-\pi^2,\ \pi \sin \pi x + \cos \pi x$
3. $-4\pi^2,\ \pi \sin \pi x + \pi \cos 2\pi x$
4. $-\pi^2,\ \pi \cos \pi x + \sin \pi x$

101. The integral equation

$\phi(x) - \frac{2}{\pi}\int_0^{\pi} \cos(x+t)\phi(t)dt = f(x)$

has infinitely many solutions if

1. $f(x) = \cos x$
2. $f(x) = \cos 3x$
3. $f(x) = \sin x$
4. $f(x) = \sin 3x$

102. Which of the following are canonical transformations? (Where q, p represent generalized coordinate and generalised momentum respectively)

1. $P = \log \sin p,\quad Q = q \tan p$
2. $P = qp^2,\quad Q = \frac{1}{p}$
3. $P = q \cot p,\quad Q = \log\left(\frac{1}{q}\sin p\right)$
4. $P = q^2 \sin 2p,\quad Q = q^2 \cos 2p$

Unit-4

103. A fair die is thrown two times independently. Let X, Y be the outcomes of these two throws and $Z = X + Y$. Let U be the remainder obtained when Z is divided by 6. Then which of the following statement(s) is/are true?

1. X and Z are independent
2. X and U are independent
3. Z and U are independent
4. Y and Z are not independent

104. A and B play a game of tossing a fair coin. A starts the game by tossing the coin once and B then tosses the coin twice, followed by A tossing the coin once and B tossing the coin twice and this continues until a head turns up. Whoever gets the first head wins the game. Then,

1. $P(B \text{ Wins}) > P(A \text{ Wins})$
2. $P(B \text{ Wins}) = 2P(A \text{ Wins})$
3. $P(A \text{ Wins}) > P(B \text{ Wins})$
4. $P(A \text{ Wins}) = 1 - P(B \text{ Wins})$

105. Consider the Markov Chain with state space $S = \{1, 2, \dots, n\}$ where $n > 10$. Suppose that the transition probability matrix $P = ((p_{ij}))$ satisfies

$p_{ij} > 0$ if $|i - j|$ is even
$p_{ij} = 0$ if $|i - j|$ is odd.

Then

1. The Markov chain is irreducible.
2. There exists a state i which is transient.
3. There exists a state i with period $d(i) = 1$.
4. There are infinitely many stationary distributions.

106. Let $\{X_i\,;\, i \ge 1\}$ be a sequence of independent random variables each having a normal distribution with mean 2 and variance 5. Then which of the following are true

1. $\frac{1}{n}\sum_{i=1}^{n} X_i$ converges in probability to 2.
2. $\frac{1}{n}\sum_{i=1}^{n} X_i^2$ converges in probability to 9.
3. $\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right)^2$ converges in probability to 4.
4. $\sum_{i=1}^{n}\left(\frac{X_i}{n}\right)^2$ converges in probability to 0.

107. Let X be a random variable with a certain non-degenerate distribution. Then identify the correct statements

1. If X has an exponential distribution then median $(X) < E(X)$
2. If X has a uniform distribution on an interval $[a, b]$, then $E(X) <$ median (X).
3. If X has a Binomial distribution then $V(X) < E(X)$
4. If X has a normal distribution, then $E(X) < V(X)$

108. Suppose the probability mass function of a random variable X under the parameter $\theta = \theta_0$ and $\theta = \theta_1 (\neq \theta_0)$ are given by

x	0	1	2	3
$p_{\theta_0}(x)$	0.01	0.04	0.5	0.45
$p_{\theta_1}(x)$	0.02	0.08	0.4	0.5

Define a test ϕ such that

$$\phi(x) = 1 \quad \text{if } x = 0, 1$$
$$= 0 \quad \text{if } x = 2, 3$$

For testing $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$, the test ϕ is

1. a most powerful test at level 0.05
2. a likelihood ratio test at level 0.05
3. an unbiased test
4. test of size 0.05

109. $X_1, X_2, \ldots, X_n$ are independent and identically distributed as $N(\mu, \sigma^2), -\infty < \mu < \infty,\ \sigma^2 > 0$. Then

1. $\sum_1^n \frac{(X_i - \bar{X})^2}{n-1}$ is the Minimum Variance Unbiased Estimate of σ^2
2. $\sqrt{\sum_1^n \frac{(X_i - \bar{X})^2}{n-1}}$ is the Minimum Variance Unbiased Estimate of σ
3. $\sum_1^n \frac{(X_i - \bar{X})^2}{n}$ is the Maximum Likelihood Estimate of σ^2
4. $\sqrt{\sum_1^n \frac{(X_i - \bar{X})^2}{n}}$ is the Maximum Likelihood Estimate of σ

110. Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed random variables each following uniform $(1-\theta, 1+\theta)$ distribution, $\theta > 0$. Define

$X_{(1)} = min\{X_1, X_2, \ldots X_n\}$,

$X_{(n)} = max\{X_1, X_2, \ldots, X_n\}$ and $\bar{X} = \frac{1}{n}\sum_{i=1}^n X_i$.

Which of the following is true?

1. $(X_{(1)}, \bar{X}, X_{(n)})$ is sufficient for θ
2. $\frac{1}{2}(X_{(n)} - X_{(1)})$ is unbiased for θ
3. $\frac{3}{n}\sum_{i=1}^n (X_i - 1)^2$ is unbiased for θ^2
4. $\frac{3}{n}\sum_{i=1}^n (X_i - \bar{X})^2$ is unbiased for θ^2

111. Suppose $X_1, X_2, \ldots, X_m$ are independent and identically distributed with common continuous distribution function $F(x)$ and $Y_1, Y_2, \ldots, Y_n$ are independent and identically distributed with common continuous distribution function $F(x - \theta)$. Also suppose X_i and Y_j are independent for all i, j. Consider the problem of testing $H_0: \theta = 0$ against $H_1: \theta > 0$.

Let $R_\alpha = Rank\ (X_\alpha), \alpha = 1, 2, \ldots, m$ and $R_{m+\beta} = Rank\ (Y_\beta),\ \beta = 1, 2, \ldots, n$ among $X_1, X_2, \ldots, X_m, Y_1, Y_2, \ldots, Y_n$.

Define $U = \sum_{\alpha=1}^m \sum_{\beta=1}^n \psi(X_\alpha, Y_\beta)$, where

$$\psi(a, b) = 1 \ \text{if } a < b,$$
$$= 0 \ \text{if } a \geq b.$$

Which of the following are true?

1. $P[R_1 = n+m, R_2 = n+m-1, R_3 = n+m-2, \ldots, R_{m+n} = 1] = \frac{1}{(m+n)!}$ under H_0
2. U and $\sum_{\alpha=1}^m R_\alpha$ are linearly related
3. $E(U) = \frac{mn}{2}$ under H_0
4. Right tailed test based on U is appropriate for testing H_0 against H_1

112. θ is the probability of obtaining a head in the toss of a coin. The coin is tossed three times and we record

$Y = 1$ if all the three tosses result in heads
$Y = 2$ if all the three tosses result in tails
$Y = 3$ otherwise

If the prior density of θ is Beta (α, β), and $\hat{\theta}_i$ is the posterior mean of θ given $Y = i$ for $i = 1, 2$, then

1. $\hat{\theta}_1 > \hat{\theta}_2$
2. $\hat{\theta}_1 < \hat{\theta}_2$
3. The posterior density of θ given $Y = 3$ is a Beta density
4. The posterior density of θ given $Y = 3$ is not a Beta density

113. Suppose $X_1, X_2, \dots, X_k$ are independent and identically distributed standard normal random variables, and $\underline{X} = (X_1, X_2, \dots, X_k)^T$. If A is an idempotent $k \times k$ matrix, then which of the following statements are true?

1. $\underline{X}^T A \underline{X}$ and $\underline{X}^T (I - A)\underline{X}$ are independent.
2. $\underline{X}^T A \underline{X}$ and $\underline{X}^T (I - A)\underline{X}$ are identically distributed if k is even and trace $(A) = \frac{k}{2}$.
3. $\frac{1}{2}\underline{X}^T A \underline{X}$ follows a gamma distribution if $A \neq 0$.
4. $\underline{X}^T (I - A)\underline{X}$ follows a chi-squared distribution if $A \neq I$.

114. For a data set $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ the following two models were fitted using least square method.

Model 1:

$y_i = \beta_0 + \beta_1 x_i \qquad i = 1, 2, \dots n$

Model 2:

$y_i = \beta_0 + \beta_1 x_i + \beta_2 {x_i}^2 \qquad i = 1, 2, \dots n$

Let $\hat{\beta}_0, \hat{\beta}_1$ be least square estimates of β_0, β_1 from model 1 and $\beta_0^*, \beta_1^*, \beta_2^*$ be the least square estimates from model 2.

Let $A = \sum_1^n \left(Y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i)\right)^2$,

$B = \sum_1^n \left(Y_i - (\beta_0^* + \beta_1^* x_i + \beta_2^* {x_i}^2)\right)^2$

Then

1. $A \geq B$
2. $A \leq B$
3. It can happen that $A = 0$ but $B > 0$
4. It can happen that $B = 0$ but $A > 0$

115. Let $\phi(x, y; \rho)$ be the density of bivariate normal distribution with mean vector $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and Variance-Covariance matrix $\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$.

Consider a random vector $\begin{pmatrix} X \\ Y \end{pmatrix}$ having density $\frac{1}{2}\left(\phi\left((x, y; \frac{1}{2}\right) + \phi\left((x, y; -\frac{1}{2}\right)\right)$. Then

1. Marginal distribution of both X and Y is standard normal.
2. $Covariance\ (X, Y) = 0$
3. X and Y are independent.
4. (X, Y) has a bivariate normal distribution.

116. Suppose $\bar{Y}$ is the sample mean of the study variables corresponding to a sample of size n using simple random sampling with replacement scheme and $\bar{Y}_{St}$ is the sample mean of the study variables corresponding to a sample of size n using stratified random sampling with replacement scheme under proportional allocation. Which of the following is/are sufficient condition/conditions for $Var(\bar{Y}) = Var(\bar{Y}_{St})$?

1. All the stratum sizes are equal
2. All the stratum totals are equal
3. All the stratum means are equal
4. All the stratum variances are equal

117. We are given some balanced incomplete block designs (BIBDs) with parameters (v, b, r, k, λ) such that $\lambda = 1$ and $k = 1$ (are fixed). With which of the following values of v can one construct such a BIBD?

1. $v = 15$
2. $v = 23$
3. $v = 25$
4. $v = 28$

118. A data set gave a 95% confidence interval (2.5, 3.6), for the mean μ of a normal population with known variance. Let $\mu_0 < 2.5$ be a fixed number. If we use the same data to test $H_0: \mu = \mu_0 \quad H_1: \mu \neq \mu_0$

1. H_0 would be necessarily rejected at $\alpha = .1$
2. H_0 would be necessarily rejected at $\alpha = .025$
3. For $\alpha = .1$, the information is not enough to draw a conclusion
4. For $\alpha = .025$, the information is not enough to draw a conclusion

119. Suppose T follows exponential distribution with unit mean. Which of the following statement(s) are correct?

1. The hazard function of T is a constant function.
2. The hazard function of T^2 is a constant function.
3. The hazard function of T^3 is the identity function.
4. The hazard function of $\sqrt{2T}$ is the identity function.

120. Consider the linear programming problem (LPP) maximize $z = 3x + 5y$

Subject to $x + 5y \leq 10$

$2x + 2y \leq 5$

$x \geq 0,\ y \geq 0.$

Then

1. The LPP does not admit any feasible solutions.
2. There exists a unique optimal solution to the LPP.
3. There exists a unique optimal solution to the dual problem.
4. The dual problem has an unbounded solution

FOR ROUGH WORK

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JOINT CSIR-UGC-JRF/NET DECEMBER 2016 ANSWER KEY BOOKLET A

MATHEMATICAL SCIENCE ENGLISH AND BILINGUAL

Set A	Key
1	3
2	3
3	3
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120	2,3