Sr. No. 4699

E

SUBJECT CODE: **4** BOOKLET CODE: **A**

June 2017

2017 (I)
MATHEMATICAL SCIENCES
TEST BOOKLET

Time : 3:00 Hours *Maximum Marks: 200*

INSTRUCTIONS

1. **You have opted for English as medium of Question Paper.** This Test Booklet contains one hundred and twenty (20 Part 'A'+40 Part 'B' +60 Part 'C') Multiple Choice Questions (MCQs). You are required to answer a maximum of 15, 25 and 20 questions from part 'A' 'B' and 'C' respectively. If more than required number of questions are answered, only first 15, 25 and 20 questions in Parts 'A' 'B' and 'C' respectively, will be taken up for evaluation.
2. **OMR** answer sheet has been provided separately. Before you start filling up your particulars, please ensure that the booklet contains requisite number of pages and that these are not torn or mutilated. If it is so, you may request the Invigilator to change the booklet of the same code. Likewise, check the **OMR** answer sheet also. Sheets for rough work have been appended to the test booklet.
3. Write your Roll No., Name and Serial Number of this Test Booklet on the **OMR** Answer sheet in the space provided. Also put your signatures in the space earmarked.
4. **You must darken the appropriate circles with a black ball pen related to Roll Number, Subject Code, Booklet Code and Centre Code on the OMR answer sheet. It is the sole responsibility of the candidate to meticulously follow the instructions given on the OMR Answer Sheet, failing which, the computer shall not be able to decipher the correct details which may ultimately result in loss, including rejection of the OMR answer sheet.**
5. Each question in Part 'A' carries 2 marks, Part 'B' 3 marks and Part 'C' 4.75 marks respectively. There will be negative marking @ 0.5marks in Part 'A' and @ 0.75 marks in Part 'B' for each wrong answer and no negative marking for Part 'C'.
6. Below each question in Part 'A' and 'B', four alternatives or responses are given. Only one of these alternatives is the "correct" option to the question. You have to find, for each question, the correct or the best answer. In Part 'C' each question may have '**ONE**' or '**MORE**' correct options. Credit in a question shall be given only on identification of '**ALL**' the correct options in Part 'C'.
7. Candidates found copying or resorting to any unfair means are liable to be disqualified from this and future examinations.
8. Candidate should not write anything anywhere except on **OMR** answer sheet or sheets for rough work.
9. Use of calculator is **NOT** permitted.
10. **After the test is over, at the perforation point, tear the OMR answer sheet, hand over the original OMR answer sheet to the invigilator and retain the carbonless copy for your record.**
11. Candidates who sit for the entire duration of the exam will only be permitted to carry their Test booklet.

Roll No.....................................

Name ....

I have verified all the information filled in by the candidate.

....

Signature of the Invigilator

S/08/RSC/17-4 AE—1A

PART 'A'

1. N is a two digit number such that the product of its digits when added to their sum equals N. The unit digit of N would be

1. 1
2. 7
3. 8
4. 9

2. If $P + \frac{1}{Q} = 1$ and $Q + \frac{1}{R} = 1$, then what is PQR?

1. −1
2. 2
3. −2
4. cannot be calculated

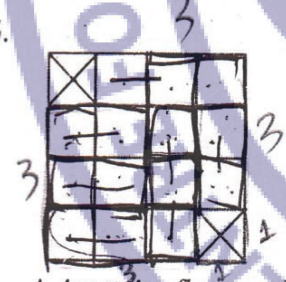
3. What is the remainder when 3^{256} is divided by 5?

1. 1
2. 2
3. 3
4. 4

4. If equal weights of 22 carat gold (alloy of 22 parts gold and 2 parts copper by weight) and 24 carat gold (pure gold) are mixed to form an alloy, what will be the weight proportion of copper in the alloy?

1. $\frac{1}{2}$
2. $\frac{1}{8}$
3. $\frac{1}{12}$
4. $\frac{1}{24}$

5.



A $4m \times 4m$ floor needs to be covered by tiles of size $2m \times 1m$. Two diagonally opposite corners of size $1m \times 1m$ should be left uncovered. How many tiles are required to complete the job without breaking the tiles or overlapping them?

1. 6
2. 7
3. 8
4. Impossible to cover

6. If 42 → 26, 71 → 78, 33 → 16, then 62 →

1. 68
2. 54
3. 38
4. 39

A shopkeeper sells a file and a notebook for Rs 27 to the first customer, a notebook and a pen for Rs 31 to the second customer and a pen and file for Rs 29 to the third customer. The prices of the items are rounded in rupees. Which of the following inferences is correct?

1. The pen is the costliest of the three
2. The file is the costliest of the three
3. The notebook is the costliest of the three
4. The shopkeeper sold the different items to different customers at different rates.

8. Consider a square of side a. Fit the largest possible circle inside it and the largest possible square inside the circle. What is the side length of the innermost square?

1. $a/\pi\sqrt{2}$
2. $a/2$
3. $\frac{a}{2\sqrt{2}}$
4. $a/\sqrt{2}$

9. Walking from my home at a speed of 5 km/h I am 8 minutes late in reaching my office. If I walk at a speed of 8 km/h I reach 5 minutes late. How far is my office from the house?

1. 2 km
2. 1/3 km
3. 2/3 km
4. 1/2 km

10. A, B and C are three distinct digits. If they are added as below,

$$\begin{array}{cccc} & A & B & C \\ + & A & B & C \\ + & A & B & C \\ \hline & C & C & C \end{array}$$

find out the value of A, B and C

1. A = 3, B = 4, C = 5
2. A = 2, B = 3, C = 1
3. A = 5, B = 1, C = 3
4. A = 1, B = 8, C = 5

11. A tight fitting band is wrapped around the Equator. Another circular band whose length is 15 m more lies at a certain height over the first band. A group of human beings attempt to pass under the longer band. Can they walk under it? (Earth's circumference is roughly 40,000 km. The height of human beings is between 1 & 2 m)

1. Yes
2. No
3. Can not be determined
4. Only those with height less than 1.7 m

12. L is the tallest and eldest of a group of five people K, L, M, N and P. M is elder to N and shorter than K. M and P are of same age and P is taller than K. N and K are of same height and K is younger to P. Which of the following inferences is certain?

1. P is taller than M
2. N is the youngest
3. N is elder to P
4. N is elder to K

13. If the product of three consecutive positive integers is equal to their sum, then what would be the sum of their squares?

1. 9
2. 14
3. 16
4. 24

14. A tall metal cylinder is filled end-to-end with n snugly fitting spherical wax balls of diameter d. If the balls melt completely, the volume fraction occupied by the melted wax is

1. independent of both d and n
2. dependent on both d and n
3. independent of d, but dependent on n
4. dependent on d, but independent of n

15. Some fishermen caught some fish. No one caught more than 20 fish. a_1 number of fishermen caught at least one fish among them, a_2 number of fishermen caught at least two fish among them, and so on and a_{20} number of fishermen caught exactly 20 fish among them. How many fish were caught?

1. $a_1 + a_2 + a_3 + \cdots + a_{20}$
2. $a_1 + 2a_2 + 3a_3 + \cdots + 20a_{20}$
3. $20(a_1 + a_2 + a_3 + \cdots + a_{20})$
4. $20(a_1 + 2a_2 + 3a_3 + \cdots + 20a_{20})$

16. If NET14 & NET15 are five digit numbers such that their sum = 157229, then N + E + T would be

1. 15
2. 21
3. 25
4. 72

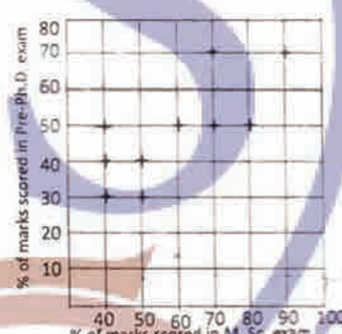
17. A cylindrical cake is to be cut into 16 equal pieces. What is the minimum number of cuts required to do so?

1. 9
2. 3
3. 8
4. 5

18. The diagram shows a cubic block of marble $(1 \times 1 \times 1\ m^3)$ having a planar fracture. What is the maximum number of slabs sized $20 \times 20 \times 5\ cm^3$ that can be cut from this block avoiding the fracture?

1. 200
2. 300
3. 400
4. 500

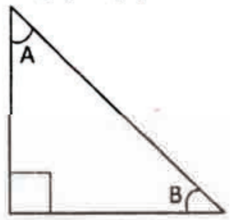
19.



Pre-Ph.D. exam score of 10 students are plotted against their M.Sc. marks. Which of the following is true?

1. Two students have scored better in Pre-Ph.D. than their M.Sc. exam.
2. All those students who scored 50% in Pre-Ph.D. scored more percentage of marks in their M.Sc. exam.
3. Two students scored the same percentage of marks in their Pre-Ph.D. and M.Sc. exams.
4. The student who scored maximum in M.Sc. is the only student to get maximum in Pre-Ph.D. exam

20. With reference to the right-angled triangle shown, what is the value of sin(A)cos(B) + cos(A)sin(B)?



1. $-1/_2$
2. 1
3. $+1/_2$
4. -1

PART 'B'

Unit-1

21. $L = \lim_{n\to\infty} \frac{1}{\sqrt[n]{n!}}$. Then

1. $L = 0$
2. $L = 1$
3. $0 < L < \infty$
4. $L = \infty$

22. Consider the sequence

$a_n = \left(1 + (-1)^n \frac{1}{n}\right)^n$

Then

1. $\limsup_{n\to\infty} a_n = \liminf_{n\to\infty} a_n = 1$
2. $\limsup_{n\to\infty} a_n = \liminf_{n\to\infty} a_n = e$
3. $\limsup_{n\to\infty} a_n = \liminf_{n\to\infty} a_n = \frac{1}{e}$
4. $\limsup_{n\to\infty} a_n = e,\ \liminf_{n\to\infty} a_n = \frac{1}{e}$

23. For $a > 0$, the series

$$\sum_{n=1}^{\infty} a^{\ln n}$$

is convergent if and only if

1. $0 < a < e$
2. $0 < a \leq e$
3. $0 < a < \frac{1}{e}$
4. $0 < a \leq \frac{1}{e}$

24. Let $f: \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} \frac{\sin x}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

Then

1. f is not continuous
2. f is continuous but not differentiable
3. f is differentiable
4. f is not bounded

25. Let

$A = \{n \in \mathbb{N}: n =$ 1 or the only prime factors of n are 2 or 3$\}$, for example, $6 \in A,\ 10 \notin A$.

Let $S = \sum_{n\in A} \frac{1}{n}$. Then

1. A is finite
2. S is a divergent series
3. $S = 3$
4. $S = 6$

26. For $n \geq 1$, let $f_n(x) = xe^{-nx^2},\ x \in \mathbb{R}$. Then the sequence $\{f_n\}$ is

1. uniformly convergent on $\mathbb{R}$
2. uniformly convergent only on compact subsets of $\mathbb{R}$
3. bounded and not uniformly convergent on $\mathbb{R}$
4. a sequence of unbounded functions

27. Let A be a 4×4 matrix. Suppose that the null space $N(A)$ of A is

$\{(x, y, z, w) \in \mathbb{R}^4:\ x + y + z = 0,\ x + y + w = 0\}$. Then

1. dim (column space(A)) = 1
2. dim (column space(A)) = 2
3. rank (A) = 1
4. $S = \{(1, 1, 1, 0), (1, 1, 0, 1)\}$ is a basis of $N(A)$

28. Let A and B be real invertible matrices such that $AB = -BA$. Then

1. Trace (A) = Trace $(B) = 0$
2. Trace (A) = Trace $(B) = 1$
3. Trace $(A) = 0$, Trace $(B) = 1$
4. Trace $(A) = 1$, Trace $(B) = 0$

29. Let A be an $n \times n$ self-adjoint matrix with eigenvalues $\lambda_1, \cdots, \lambda_n$.
Let $\|X\|_2 = \sqrt{|x_1|^2 + \cdots + |x_n|^2}$ for $X = (x_1, \cdots, x_n) \in \mathbb{C}^n$.
If $p(A) = a_0 I + a_1 A + \cdots + a_n A^n$ then $\sup_{\|X\|_2=1} \|p(A)X\|_2$ is equal to

1. $\max\{a_0 + a_1\lambda_j + \cdots + a_n\lambda_j^n : 1 \le j \le n\}$
2. $\max\{|a_0 + a_1\lambda_j + \cdots + a_n\lambda_j^n| : 1 \le j \le n\}$
3. $\min\{a_0 + a_1\lambda_j + \cdots + a_n\lambda_j^n : 1 \le j \le n\}$
4. $\min\{|a_0 + a_1\lambda_j + \cdots + a_n\lambda_j^n| : 1 \le j \le n\}$

30. Let $p(x) = \alpha x^2 + \beta x + \gamma$ be a polynomial, where $\alpha, \beta, \gamma \in \mathbb{R}$. Fix $x_0 \in \mathbb{R}$. Let
$S = \{(a, b, c) \in \mathbb{R}^3 : p(x) = a(x - x_0)^2 + b(x - x_0) + c \text{ for all } x \in \mathbb{R}\}$.
Then the number of elements in S is

1. 0
2. 1
3. strictly greater than 1 but finite
4. infinite

31. Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$ and I be the 3×3 identity matrix.
If $6A^{-1} = aA^2 + bA + cI$ for $a, b, c \in \mathbb{R}$ then (a, b, c) equals

1. (1, 2, 1)
2. (1, −1, 2)
3. (4, 1, 1)
4. (1, 4, 1)

32. Let $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & -2 & 5 \\ 2 & 5 & -3 \end{bmatrix}$. Then the eigenvalues of A are

1. $-4, 3, -3$. ✓
2. $4, 3, 1$
3. $4, -4 \pm \sqrt{13}$
4. $4, -2 \pm 2\sqrt{7}$

Unit-2

33. Let C denote the unit circle centered at the origin in $\mathbb{C}$.

Then $\frac{1}{2\pi i}\int_C |1 + z + z^2|^2 dz$,

where the integral is taken anti-clockwise along C, equals

1. 0
2. 1
3. 2
4. 3

34. Consider the power series

$f(x) = \sum_{n=2}^{\infty} \log(n) x^n$.

The radius of convergence of the series $f(x)$ is

1. 0
2. 1
3. 3
4. ∞

35. For an odd integer $k \ge 1$, let $\mathcal{F}$ be the set of all entire functions f such that

$f(x) = |x^k|$ for all $x \in (-1, 1)$. Then the cardinality of $\mathcal{F}$ is

1. 0
2. 1
3. strictly greater than 1 but finite
4. infinite

36. Suppose f is holomorphic in an open neighbourhood of $z_0 \in \mathbb{C}$. Given that the series

$\sum_{n=0}^{\infty} f^{(n)}(z_0)$

converges absolutely, we can conclude that

1. f is constant
2. f is a polynomial
3. f can be extended to an entire function
4. $f(x) \in \mathbb{R}$ for all $x \in \mathbb{R}$

37. Let S be the set of all integers from 100 to 999 which are neither divisible by 3 nor divisible by 5. The number of elements in S is

1. 480
2. 420
3. 360
4. 240

38. The remainder obtained when 16^{2016} is divided by 9 equals

1. 1
2. 2
3. 3
4. 7

39. Consider the ideal $I = (x^2 + 1, y)$ in the polynomial ring $\mathbb{C}[x, y]$. Which of the following statements is true?

1. I is a maximal ideal
2. I is a prime ideal but not a maximal ideal
3. I is a maximal ideal but not a prime ideal
4. I is neither a prime ideal nor a maximal ideal

40. Let $f: \mathbb{R} \to \mathbb{R}$ be a continuous map. Choose the correct statement.

1. f is bounded
2. The image of f is an open subset of $\mathbb{R}$
3. $f(A)$ is bounded for all bounded subsets A of $\mathbb{R}$
4. $f^{-1}(A)$ is compact for all compact subsets A of $\mathbb{R}$

Unit-3

41. Suppose $x: [0, \infty) \to [0, \infty)$ is continuous and $x(0) = 0$.

If $(x(t))^2 \leq 2 + \int_0^t x(s)ds, \quad \forall t \geq 0$, then which of the following is TRUE?

1. $x(\sqrt{2}) \in [0,2]$
2. $x(\sqrt{2}) \in \left[0, \frac{3}{\sqrt{2}}\right]$
3. $x(\sqrt{2}) \in \left[\frac{5}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right]$
4. $x(\sqrt{2}) \in [10, \infty)$

42. The solution of the partial differential equation

$$u_t - xu_x + 1 - u = 0, \quad x \in \mathbb{R}, \quad t > 0$$

subject to $u(x, 0) = g(x)$ is

1. $u(x,t) = 1 - e^{-t}(1 - g(xe^t))$
2. $u(x,t) = 1 + e^t(1 - g(xe^t))$
3. $u(x,t) = 1 - e^{-t}(1 - g(xe^{-t}))$
4. $u(x,t) = e^{-t}(1 - g(xe^t))$

43. Suppose $u \in C^2(\bar{B})$, B is the unit ball in $\mathbb{R}^2$, satisfies

$$\Delta u = f \text{ in } B$$

$$\alpha u + \frac{\partial u}{\partial n} = g \quad \text{on } \partial B, \quad \alpha > 0,$$

where n is the unit outward normal to B. If a solution exists then

1. it is unique
2. there are exactly two solutions
3. there are exactly three solutions
4. there are infinitely many solutions

44. The magnitude of the truncation error for the scheme

$$f'(x) = Af(x) + Bf(x+h) + Cf(x+2h)$$

is equal to

1. $h^2 f'''(\xi)$ if $A = -\frac{5}{6h}$, $B = \frac{3}{2h}$, $C = -\frac{2}{3h}$.
2. $h^2 f'''(\xi)$ if $A = \frac{5}{6h}$, $B = \frac{3}{2h}$, $C = \frac{2}{3h}$.
3. $h^2 f''(x)$ if $A = -\frac{5}{6h}$, $B = \frac{3}{2h}$, $C = -\frac{2}{3h}$.
4. $h^2 f''(x)$ if $A = \frac{5}{6h}$, $B = \frac{3}{2h}$, $C = \frac{2}{3h}$.

45. The infimum of $\int_0^1 (u'(t))^2 dt$ on the class of functions

$$\left\{u \in C^1[0,1] \text{ such that } u(0) = 0 \text{ and } \max_{[0,1]}|u| = 1\right\}$$

is equal to

1. 0
2. 1/2
3. 1
4. 2

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46. Let $\phi(x)$ be the solution of $\int_0^x e^{x-t}\phi(t)dt = x, \quad x > 0$. Then $\phi(1)$ equals

1. -1
2. 0
3. 1
4. 2

47. A rigid body having one point O fixed and no external torque about O has equal principal moments of inertia. Then the body must rotate with

1. angular velocity of variable magnitude
2. angular velocity with constant magnitude
3. constant angular momentum but varying angular velocity
4. varying angular momentum with varying angular velocity

48. Consider a spherical pendulum consisting of a particle of mass m which moves under gravity on a smooth sphere of radius a. In terms of spherical polar angles θ, ϕ, with θ measured up from the downward vertical, the Lagrangian is given by

1. $ma\left[\frac{a}{2}(\dot\theta^2 + \dot\phi^2 \sin^2\theta) - g\cos\theta\right]$
2. $ma\left[\frac{a}{2}(\dot\theta^2 + \dot\phi^2 \sin^2\theta) + g\cos\theta\right]$
3. $ma\left[\frac{a}{2}(\dot\theta^2 + \dot\phi^2 \cos^2\theta) + g\sin\theta\right]$
4. $ma\left[\frac{a}{2}(\dot\theta^2 + \dot\phi^2 \cos^2\theta) - g\sin\theta\right]$

40 R 60 B

Unit-4

49. A box contains 40 numbered red balls and 60 numbered black balls. From the box, balls are drawn one by one at random without replacement till all the balls are drawn. The probability that the last ball drawn is black equals

1. $1/100$
2. $1/60$
3. $3/5$
4. $2/3$

50. $X_1, X_2, \cdots$ are independent identically distributed random variables having common density f. Assume $f(x) = f(-x)$ for all $x \in \mathbb{R}$. Which of the following statements is correct?

1. $\frac{1}{n}(X_1 + \cdots + X_n) \to 0$ in probability
2. $\frac{1}{n}(X_1 + \cdots + X_n) \to 0$ almost surely
3. $P\left(\frac{1}{\sqrt{n}}(X_1 + \cdots + X_n) < 0\right) \to \frac{1}{2}$
4. $\sum_{i=1}^n X_i$ has the same distribution as $\sum_{i=1}^n (-1)^i X_i$

51. Let N_t denote the number of accidents up to time t. Assume that $\{N_t\}$ is a Poisson process with intensity 2. Given that there are exactly 5 accidents during the time period [20, 30], what is the conditional probability that there is exactly one accident during the time period $[15, 25]$?

1. $\frac{15}{32}e^{-10}$
2. $20\, e^{-20}$
3. $\frac{10^5}{5!}e^{-30}$
4. $\frac{1}{5}$

52. X and Y are independent random variables each having the density

$f(t) = \frac{1}{\pi}\frac{1}{1+t^2}, \quad -\infty < t < \infty.$

Then the density function of $\frac{X+Y}{3}$ for $-\infty < t < \infty$ is given by

1. $\frac{6}{\pi}\frac{1}{4+9t^2}$
2. $\frac{6}{\pi}\frac{1}{9+4t^2}$
3. $\frac{3}{\pi}\frac{1}{1+9t^2}$
4. $\frac{3}{\pi}\frac{1}{9+t^2}$

53. Suppose $\{X_1, \cdots, X_n\}, n \geq 2$, is a random sample from the distribution with probability density function

$$f(x;\theta) = \begin{cases} \frac{\theta^\theta}{\Gamma(\theta)} x^{\theta-1}e^{-x\theta} & ; \ x > 0 \\ 0 & ; \ x \leq 0 \end{cases}$$

with $\theta > 0$. Then the *method of moments* estimator of θ

1. does not exist
2. is $\frac{n}{\sum_{i=1}^n (X_i - 1)^2}$

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3. is $\frac{n}{\sum_{i=1}^{n}(X_i-\bar{X})^2}$
4. is $\frac{n-1}{\sum_{i=1}^{n}(X_i-1)^2}$

54. Let $X_1, X_2, \cdots, X_n$ for $n \geq 5$ be a random sample from the distribution with probability density function

$$f(x;\theta) = \begin{cases} e^{-(x-\theta)} & \text{if } x > \theta \\ 0 & \text{otherwise} \end{cases}$$

for $\theta > 0$. The confidence coefficient of the confidence interval $\left[\min\{X_1,\cdots,X_n\} - \frac{\ln 4}{n}, \min\{X_1,\cdots,X_n\} + \frac{\ln 2}{n}\right]$ for θ, is

1. 0.5
2. 0.75
3. 0.95
4. $1 - \frac{1}{2^n}$

55. Let X be a random sample from an exponential distribution with mean $1/\lambda$. If λ has a prior distribution with probability density function

$$g(\lambda) = \begin{cases} \lambda e^{-\lambda} & ; \lambda > 0 \\ 0 & ; \lambda \leq 0 \end{cases}$$

then the Bayes estimator of $1/\lambda$ with respect to the squared error loss function is

1. $\frac{2}{X+1}$
2. $\frac{1}{X}$
3. X
4. $\frac{X+1}{2}$

56. Consider the linear statistical model

$y_{ij} = \mu + \tau_i + \varepsilon_{ij}$; $i = 1, 2, \cdots, a$; $j = 1, 2, \cdots, n$

where μ is unknown, τ_i are independently and identically distributed as $N(0, \sigma_\tau^2)$, ε_{ij} are independently and identically distributed as $N(0, \sigma^2)$; τ_i and ε_{ij} are independent for all i and j. Note that τ_i is the i^{th} treatment effect. Suppose $SS_{total}, SS_{treatment}, SS_{error}$ are total sum of squares, total treatment sum of squares and error sum of squares, respectively.
To test $:H_0: \sigma_\tau^2 = 0$ vs $H_A: \sigma_\tau^2 > 0$ which of the following statements is **not** true?

1. The sum of squares identity is $SS_{total} = SS_{treatment} + SS_{error}$
2. $SS_{error} \sim \sigma^2 x^2{}_{n(a-1)}$
3. Under H_0, $\frac{\frac{SS_{treatment}}{a-1}}{\frac{SS_{error}}{n(a-1)}} \sim F_{a-1,n(a-1)}$
4. $E(SS_{error}) = n(a-1)(\sigma^2 + n\sigma_\tau^2)$

57. Suppose (X_1, X_2) follows a bivariate normal distribution with $E(X_1) = E(X_2) = 0$, $V(X_1) = V(X_2) = 2$ and $Cov(X_1, X_2) = -1$. If $\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-y^2/2}dy$, then $P[X_1 - X_2 > 6]$ is equal to

1. $\Phi(-1)$
2. $\Phi(-3)$
3. $\Phi(\sqrt{6})$
4. $\Phi(-\sqrt{6})$

58. Consider the problem of drawing a sample of size 2 from a finite population of size 20. The sampling is done with replacement using probability proportional to size sampling scheme. The normed size measures $p_1, \cdots, p_{20}$ are given by $p_i = \frac{1}{40}$, $i = 1, \cdots, 10$, $p_i = \frac{3}{40}$, $i = 11, \cdots, 20$.

The expected number of distinct units drawn is

1. $\frac{83}{80}$
2. $\frac{157}{80}$
3. $\frac{17}{16}$
4. $\frac{31}{16}$

59. If we interchange two columns of a Latin square design (LSD), then the new design is

1. an LSD
2. a completely randomised design (CRD) but not an LSD
3. a randomised block design (RBD) but not an LSD
4. a balanced incomplete block design (BIBD) but not an LSD

60. Consider the LPP:

Minimize $c^t x$ subject to $Ax = b, x \geq 0$, where

$A = \begin{bmatrix} 1 & 1 & 3 & 1 & 2 \\ 0 & -1 & -2 & -3 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$,

$c = (2, -1, 1, -9, 0)^t$, and

$x = (x_1, x_2, x_3, x_4, x_5)^t$.

Using the revised simplex method with current basis as $\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$, which of the following statements is correct?

1. The next entering variable is x_5
2. The solution corresponding to the current basis is optimal
3. The next entering variable is x_4
4. The next entering variable is x_3

PART 'C'

Unit-1

61. Let $\alpha = 0.10110111011110\cdots$ be a given real number written in base 10, that is, the n-th digit of α is 1, unless n is of the form $\frac{k(k+1)}{2} - 1$ in which case it is 0. Choose all the correct statements from below.

1. α is a rational number
2. α is an irrational number
3. For every integer $q \geq 2$, there exists an integer $r \geq 1$ such that $\frac{r}{q} < \alpha < \frac{r+1}{q}$.
4. α has no periodic decimal expansion.

62. For $a, b \in \mathbb{N}$, consider the sequence

$$d_n = \frac{\binom{n}{a}}{\binom{n}{b}}$$

for $n > a, b$. Which of the following statements are true? As $n \to \infty$,

1. $\{d_n\}$ converges for all values of a and b
2. $\{d_n\}$ converges if $a < b$
3. $\{d_n\}$ converges if $a = b$
4. $\{d_n\}$ converges if $a > b$

63. Let $\{a_n\}$ be a sequence of real numbers satisfying $\sum_{n=1}^{\infty} |a_n - a_{n-1}| < \infty$. Then the series $\sum_{n=0}^{\infty} a_n x^n$, $x \in \mathbb{R}$ is convergent

1. nowhere on $\mathbb{R}$
2. everywhere on $\mathbb{R}$
3. on some set containing $(-1, 1)$
4. only on $(-1, 1)$

64. Let $f(x) = \tan^{-1} x$, $x \in \mathbb{R}$. Then

1. there exists a polynomial $p(x)$ satisfying $p(x)f'(x) = 1$, for all x
2. $f^{(n)}(0) = 0$ for all positive even integers n
3. the sequence $\{f^{(n)}(0)\}$ is unbounded
4. $f^{(n)}(0) = 0$ for all n

65. Let $f_n(x) = \frac{1}{1+n^2x^2}$ for $n \in \mathbb{N}$, $x \in \mathbb{R}$.

Which of the following are true?

1. f_n converges pointwise on $[0, 1]$ to a continuous function
2. f_n converges uniformly on $[0, 1]$
3. f_n converges uniformly on $\left[\frac{1}{2}, 1\right]$
4. $\lim_{n\to\infty} \int_0^1 f_n(x)dx = \int_0^1 \left(\lim_{n\to\infty} f_n(x)\right) dx$

66. If $\lambda_n = \int_0^1 \frac{dt}{(1+t)^n}$ for $n \in \mathbb{N}$, then

1. λ_n does not exist for some n
2. λ_n exists for every n and the sequence is unbounded
3. λ_n exists for every n and the sequence is bounded
4. $\lim_{n\to\infty} (\lambda_n)^{1/n} = 1$

67. The equation

$11^x + 13^x + 17^x - 19^x = 0$ has

1. no real root
2. only one real root
3. exactly two real roots
4. more than two real roots

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68. Suppose that $f:\mathbb{R}^n \to \mathbb{R}$ is given by $f(\underline{x}) = a_1x_1^2 + a_2x_2^2 + \cdots + a_nx_n^2$, where $\underline{x} = (x_1, x_2, \cdots, x_n)$ and at least one a_j is not zero. Then we can conclude that

1. f is not everywhere differentiable
2. the gradient $(\nabla f)(\underline{x}) \neq 0$ for every $\underline{x} \in \mathbb{R}^n$
3. if $\underline{x} \in \mathbb{R}^n$ is such that $(\nabla f)(\underline{x}) = 0$ then $f(\underline{x}) = 0$
4. if $\underline{x} \in \mathbb{R}^n$ is such that $f(\underline{x}) = 0$ then $(\nabla f)(\underline{x}) = 0$

69. Let S be the set of $(\alpha, \beta) \in \mathbb{R}^2$ such that

$\frac{x^{\alpha}y^{\beta}}{\sqrt{x^2+y^2}} \to 0$ as $(x, y) \to (0, 0)$.

Then S is contained in

1. $\{(\alpha, \beta): \alpha > 0,\ \beta > 0\}$
2. $\{(\alpha, \beta): \alpha > 2,\ \beta > 2\}$
3. $\{(\alpha, \beta): \alpha + \beta > 1\}$
4. $\{(\alpha, \beta): \alpha + 4\beta > 1\}$

70. Consider the vector space V of real polynomials of degree less than or equal to n. Fix distinct real numbers $a_0, a_1, \cdots, a_k$. For $p \in V$

$$\max\{|p(a_j)| : 0 \leq j \leq k\}$$

defines a norm on V

1. only if $k < n$
2. only if $k \geq n$
3. if $k + 1 \leq n$
4. if $k \geq n + 1$

71. Let V be the vector space of polynomials of degree at most 3 in a variable x with coefficients in $\mathbb{R}$. Let $T = d/dx$ be the linear transformation of V to itself given by differentiation. Which of the following are correct?

1. T is invertible
2. 0 is an eigenvalue of T
3. There is a basis with respect to which the matrix of T is nilpotent.
4. The matrix of T with respect to the basis $\{1, 1 + x, 1 + x + x^2, 1 + x + x^2 + x^3\}$ is diagonal

72. Let m, n, r be natural numbers. Let A be an $m \times n$ matrix with real entries such that $(AA^t)^r = I$, where I is the $m \times m$ identity matrix and A^t is the transpose of the matrix A. We can conclude that

1. $m = n$
2. AA^t is invertible
3. A^tA is invertible
4. if $m = n$, then A is invertible

73. Let A be an $n \times n$ real matrix with $A^2 = A$. Then

1. the eigenvalues of A are either 0 or 1
2. A is a diagonal matrix with diagonal entries 0 or 1
3. rank (A) = trace (A)
4. rank $(I - A)$ = trace $(I - A)$

74. For any $n \times n$ matrix B, let $N(B) = \{X \in \mathbb{R}^n: BX = 0\}$ be the null space of B. Let A be a 4×4 matrix with $\dim(N(A - 2I)) = 2$, $\dim(N(A - 4I)) = 1$ and rank $(A) = 3$. Then

1. 0, 2 and 4 are eigenvalues of A
2. determinant $(A) = 0$
3. A is not diagonalizable
4. trace $(A) = 8$

75. Which of the following 3×3 matrices are diagonalizable over $\mathbb{R}$?

1. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$
2. $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
3. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$
4. $\begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

76. Let H be a real Hilbert space and $M \subseteq H$ be a closed linear subspace. Let $x_0 \in H \setminus M$. Let $y_0 \in M$ be such that

$\|x_0 - y_0\| = \inf\{\|x_0 - y\| : y \in M\}$.

Then

1. such a y_0 is unique
2. $x_0 \perp M$
3. $y_0 \perp M$
4. $x_0 - y_0 \perp M$

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77. Let $A = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ and

$Q(X) = X^t AX$ for $X \in \mathbb{R}^3$. Then

1. A has exactly two positive eigenvalues
2. all the eigenvalues of A are positive
3. $Q(X) \geq 0$ for all $X \in \mathbb{R}^3$
4. $Q(X) < 0$ for some $X \in \mathbb{R}^3$

78. Consider the matrix

$$A(x) = \begin{pmatrix} 1+x^2 & 7 & 11 \\ 3x & 2x & 4 \\ 8x & 17 & 13 \end{pmatrix}; \; x \in \mathbb{R}.$$

Then

1. $A(x)$ has eigenvalue 0 for some $x \in \mathbb{R}$
2. 0 is not an eigenvalue of $A(x)$ for any $x \in \mathbb{R}$
3. $A(x)$ has eigenvalue 0 for all $x \in \mathbb{R}$
4. $A(x)$ is invertible for every $x \in \mathbb{R}$

Unit-2

79. Let $f = u + iv$ be an entire function where u, v are the real and imaginary parts of f respectively. If the Jacobian matrix

$$J_a = \begin{bmatrix} u_x(a) & u_y(a) \\ v_x(a) & v_y(a) \end{bmatrix}$$

is symmetric for all $a \in \mathbb{C}$, then

1. f is a polynomial.
2. f is a polynomial of degree ≤ 1.
3. f is necessarily a constant function.
4. f is a polynomial of degree strictly greater than 1.

80. Consider the function $f(z) = \frac{\sin(\pi z/2)}{\sin(\pi z)}$. Then f has poles at

1. all integers
2. all even integers
3. all odd integers
4. all integers of the form $4k+1$, $k \in \mathbb{Z}$

81. Consider the Möbius transformation $f(z) = \frac{1}{z}$, $z \in \mathbb{C}$, $z \neq 0$. If C denotes a circle with positive radius passing through the origin, then f maps $C\setminus\{0\}$ to

1. a circle.
2. a line.
3. a line passing through the origin.
4. a line not passing through the origin.

82. For which among the following functions $f(z)$ defined on $G = \mathbb{C}\setminus\{0\}$, is there no sequence of polynomials approximating $f(z)$ uniformly on compact subsets of G?

1. $\exp(z)$
2. $1/z$
3. z^2
4. $1/z^2$

83. For an integer $n \geq 2$, let S_n be the permutation group on n letters and A_n the alternating group. Let $\mathbb{C}^*$ be the group of non-zero complex numbers under multiplication. Which of the following are correct statements?

1. For every integer $n \geq 2$, there is a nontrivial homomorphism $\chi: S_n \to \mathbb{C}^*$.
2. For every integer $n \geq 2$, there is a unique nontrivial homomorphism $\chi: S_n \to \mathbb{C}^*$.
3. For every integer $n \geq 3$, there is a nontrivial homomorphism $\chi: A_n \to \mathbb{C}^*$.
4. For every integer $n \geq 5$, there is no nontrivial homomorphism $\chi: A_n \to \mathbb{C}^*$.

84. Let $R = \{f: \{1,2,\ldots,10\} \to \mathbb{Z}_2\}$ be the set of all $\mathbb{Z}_2$-valued functions on the set $\{1,2,\ldots,10\}$ of the first ten positive integers. Then R is commutative ring with pointwise addition and pointwise multiplication of functions. Which of the following statements are correct?

1. R has a unique maximal ideal.
2. Every prime ideal of R is also maximal.
3. Number of proper ideals of R is 511.
4. Every element of R is idempotent.

85. Which of the following rings are principal ideal domains (PID)?

1. $\mathbb{Q}[x]$
2. $\mathbb{Z}[x]$
3. $(\mathbb{Z}/6\mathbb{Z})[x]$
4. $(\mathbb{Z}/7\mathbb{Z})[x]$

86. Let G be a group of order 125. Which of the following statements are necessarily true?

1. G has a non-trivial abelian subgroup
2. The centre of G is a proper subgroup

3. The centre of G has order 5
4. There is a subgroup of order 25

87. Let R be a non-zero ring with identity such that $a^2 = a$ for all $a \in R$. Which of the following statements are true?

1. There is no such ring
2. $2a = 0$ for all $a \in R$
3. $3a = 0$ for all $a \in R$
4. $\mathbb{Z}/2\mathbb{Z}$ is a subring of R

88. Which of the following polynomials are irreducible in $\mathbb{Z}[x]$?

1. $x^4 + 10x + 5$
2. $x^3 - 2x + 1$
3. $x^4 + x^2 + 1$
4. $x^3 + x + 1$

89. Let X be any topological space. Let $A \subseteq X$ be nonempty. For $x, y \in A$, define $x \sim y$ if there is a connected subset $C \subseteq A$ such that $x, y \in C$. For $x \in A$, define $C(x) = \{y \in A : y \sim x\}$. Then

1. $C(x) = C(y) \Rightarrow x = y$
2. $C(x) = C(y) \Rightarrow x \sim y$
3. $C(x) \cap C(y) \neq \emptyset \Rightarrow x \sim y$
4. $C(x) \cap C(y) \neq \emptyset \Rightarrow C(x) = C(y)$

90. Let X be a topological space and Y a subset of X. Write $i: Y \to X$ for the inclusion map. Choose the correct statement(s):

1. If Y has the subspace topology, then i is continuous
2. If i is continuous, then Y has the subspace topology
3. If Y is an open subset of X, then $i(U)$ is open in X for all subsets $U \subseteq Y$ that are open in the subspace topology on Y
4. If Y is a compact subset of X, then $i(U)$ is open in X for all subsets $U \subseteq Y$ that are open in the subspace topology on Y

Unit-3

91. Consider the solution of the ordinary differential equation

$y'(t) = -y^3 + y^2 + 2y$ subject to $y(0) = y_0 \in (0,2)$. Then

$\lim_{t\to\infty} y(t)$ belongs to

1. $\{-1, 0\}$
2. $\{-1, 2\}$
3. $\{0, 2\}$
4. $\{0, +\infty\}$

92. If the solution to

$$\begin{cases} \frac{du}{dx} = y^2 + x^2 & , x > 0 \\ y(0) = 2 \end{cases}$$

exists in the interval $[0, L_0)$ and the maximal interval of existence of

$$\begin{cases} \frac{dz}{dx} = z^2 & , x > 0 \\ z(0) = 1 \end{cases}$$

is $[0, L_1)$, then which of the following statements are correct?

1. $L_1 = 1, \quad L_0 > 1$
2. $L_1 = 1, \quad L_0 \leq 1$
3. $L_1 < 2, \quad L_0 \leq 1$
4. $L_1 > 2, \quad L_0 < 1$

93. Consider the partial differential equation $x\frac{\partial u}{\partial x} + yu\frac{\partial u}{\partial y} = -xy$ for $x > 0$ subject to $u = 5$ on $xy = 1$. Then

1. $u(x, y)$ exists when $xy \leq 19$ and $u(x, y) = u(y, x)$ for $x > 0, y > 0$
2. $u(x, y)$ exists when $xy \geq 19$ and $u(x, y) = u(y, x)$ for $x > 0, y > 0$
3. $u(1, 11) = 3,\ u(13, -1) = 7$
4. $u(1, -1) = 5,\ u(11, 1) = -5$

94. If a complete integral of the partial differential equation

$$x(p^2+q^2)=zp; \quad p=\frac{\partial z}{\partial x},\ q=\frac{\partial z}{\partial y}$$

passes through the curve $x=0,\ z^2=4y$, then the envelope of this family passing through $x=1$ and $y=1$ has

1. $z=-2$
2. $z=2$
3. $z=\sqrt{2+2\sqrt{2}}$
4. $z=-\sqrt{2+2\sqrt{2}}$

95. For a differentiable function $f:\mathbb{R}\to\mathbb{R}$ define the difference quotient

$$(D_xf)(h)=\frac{f(x+h)-f(x)}{h}\ ;\quad h>0.$$

Consider numbers of the form $\hat{h}=h(1+\epsilon)$ for a fixed $\epsilon>0$ and let

$e_1(h)=f'(x)-(D_xf)(h),$

$e_2(h)=(D_xf)(h)-(D_xf)(\hat{h}),$

$e(h)=e_1(h)+e_2(h).$

If $f(x+\hat{h})=f(x+h)$, then

1. $e_1(h)\to 0$ as $h\to 0$.
2. $e_2(h)\to 0$ as $h\to 0$.
3. $e_2(h)\to \epsilon f'(x)/(1+\epsilon)$ as $h\to 0$.
4. $e(h)\to 0$ as $h\to 0$.

96. Let y_n satisfy $y_n=y_{n-1}+hy_{n-1}$ with $y_0=1$ $(n=1,2\ldots,N)$ and for $0<h<1,\ Nh=1$. Then

1. $y_N\to e$ as $N\to\infty$
2. $y_N\to e^h$ as $N\to\infty$
3. $y_n=(1+h)^n$
4. $y_n\geq 1$

97. Let $y(x)$ be the solution of the integral equation

$$y(x)=x-\int_0^x xt^2y(t)dt,\ x>0.$$

Then the value of the function $y(x)$ at $x=\sqrt{2}$ is equal to

1. $\frac{1}{\sqrt{2e}}$
2. $\frac{e}{2}$
3. $\frac{\sqrt{2}}{e^2}$
4. $\frac{\sqrt{2}}{e}$

98. The solutions for $\lambda=-1$ and $\lambda=3$ of the integral equation

$y(x)=1+\lambda\int_0^1 K(x,t)y(t)dt$, where

$$K(x,t)=\begin{cases}\cosh x\sinh t, & 0\leq x\leq t\\ \cosh t\sinh x, & t\leq x\leq 1\end{cases}$$

are , respectively,

1. $-\frac{x^2}{2}+\frac{3}{2}-\tanh 1$ and $\frac{1}{4}\left(\frac{3\cos 2x}{\cos 2-2\sin 2\tanh 1}+1\right)$
2. $-\frac{x^2}{2}+\frac{3}{2}-\tanh 1$ and $\frac{1}{4}\left(\frac{3\cosh 2x}{\cosh 2-2\sinh 2\tanh 1}+1\right)$
3. $\frac{x^2}{2}+\frac{3}{2}-\tanh 1$ and $\frac{1}{4}\left(\frac{3\cosh 2x}{\cosh 2-2\sinh 2\tanh 1}-1\right)$
4. $\frac{x^2}{2}+\frac{3}{2}-\tanh 1$ and $\frac{1}{4}\left(\frac{3\cos 2x}{\cos 2-2\sin 2\tanh 1}-1\right)$

99. Consider the functional

$$I(y(x))=\int_{x_0}^{x_1} f(x,y)\sqrt{1+y'^2}\,e^{\tan^{-1}y'}\,dx$$

where

$f(x,y)\neq 0$. Let the left end of the extremal be fixed at the point $A(x_0,y_0)$ and the right end $B(x_1,y_1)$ be movable along the curve $y=\psi(x)$. Then the extremal $y=y(x)$ intersects the curve $y=\psi(x)$ along which the boundary point $B(x_1,y_1)$ slides at an angle

1. $\pi/3$
2. $\pi/2$
3. $\pi/4$
4. $\pi/6$

100. Let B be the unit ball in $\mathbb{R}^2$. Let $u \in C^2(\bar{B})$ be a minimizer of

$$I(u) = \int_B (|\nabla u|^2 + fu)\,dx + \int_{\partial B} a\,u^2 ds$$

where f and a are continuous functions in $C^2(\bar{B})$. Let $\vec{n}$ denote the unit outward normal. Which of the following are correct?

1. $-2\,\Delta u + f = 0$ in B and $\frac{\partial u}{\partial \vec{n}} + a\,u = 0$ on ∂B
2. $-2\,\Delta u + f + a = 0$ in B and $\frac{\partial u}{\partial \vec{n}} = 0$ on ∂B
3. $-\Delta u + f = 0$ in B and $2\frac{\partial u}{\partial \vec{n}} + a\,u = 0$ on ∂B
4. $-\Delta u + 2\,f = 0$ in B and $2\,\frac{\partial u}{\partial \vec{n}} + a\,u = 0$ on ∂B

101. Let q_α and p_α $(\alpha = 1, 2, \dots, n)$ be the generalized coordinates and the generalized momenta, respectively. If H denotes the Hamiltonian and q_α (for some $\alpha = \alpha_0$) is an ignorable coordinate, then which of the following equations are satisfied?

1. $\dot{p}_\alpha = -\frac{\partial H}{\partial q_\alpha}, \dot{q}_\alpha = \frac{\partial H}{\partial p_\alpha}, \forall\, \alpha$
2. $\dot{p}_\alpha = \frac{\partial H}{\partial q_\alpha}, \quad \dot{q}_\alpha = -\frac{\partial H}{\partial p_\alpha}\ \forall\, \alpha$
3. $\dot{p}_{\alpha_0} = 0, \quad \dot{q}_{\alpha_0} = \frac{\partial H}{\partial p_{\alpha_0}}$
4. $\dot{p}_{\alpha_0} = -\frac{\partial H}{\partial q_{\alpha_0}}, \quad \dot{q}_{\alpha_0} = 0$

102. For a conservative system, the end configurations are fixed and the velocity in the varied motion is such that $T + V = E$. Here T, V and E represent, respectively the kinetic energy, the potential energy and the total energy. If $\delta(A)$ denotes the infinitesimal change in a variable A, and p_α and q_α $(\alpha = 1, 2, \dots, n)$ represent the generalized momenta and generalized coordinates, respectively, then

1. $\delta \int T\,dt = 0$
2. $\delta \int \sum_{\alpha=1}^{n} p_\alpha dq_\alpha = 0$
3. $\delta \int \sum_{\alpha=1}^{n} q_\alpha dp_\alpha = 0$
4. $\delta \int \sum_{\alpha=1}^{n} (p_\alpha dq_\alpha + q_\alpha dp_\alpha) = 0$

Unit-4

103. Let $c \in \mathbb{R}$ be a constant. Let X, Y be random variables with joint probability density function

$$f(x,y) = \begin{cases} cxy, & \text{if } 0 < x < y < 1, \\ 0, & \text{otherwise} \end{cases}$$

Which of the following statements are correct?

1. $c = \frac{1}{8}$
2. $c = 8$
3. X and Y are independent
4. $P(X = Y) = 0$

104. Let $\{X_n, n \geq 1\}$ be i.i.d. uniform $(-1,\ 2)$ random variables. Which of the following statements are true?

1. $\frac{1}{n}\sum_{i=1}^{n} X_i \to 0$ almost surely
2. $\left\{\frac{1}{2n}\sum_{i=1}^{n} X_{2i} - \frac{1}{2n}\sum_{i=1}^{n} X_{2i-1}\right\} \to 0$ almost surely
3. $\sup\{X_1, X_2, \dots\} = 2$ almost surely
4. $\inf\{X_1, X_2, \dots\} = -1$ almost surely

105. Let $\{X_n\}$ be a Markov chain on $\{0,1,2,\dots\}$ with

$P_{00} = \frac{2}{3}, P_{01} = \frac{1}{3}, P_{i,i+1} = \frac{2}{3}, P_{i,i-1} = \frac{1}{3}$, $i \geq 1, P_{ij} = 0$ otherwise.

Which of the following statements are correct?

1. $\{X_n\}$ is recurrent
2. $\{X_n\}$ is transient
3. $P(\lim_{n\to\infty} X_n = 0) > 0$
4. $P(\lim_{n\to\infty} X_n = +\infty) > 0$

106. Which of the following statements are correct?

1. For a finite state Markov chain there is at least one transient state.
2. For a finite state Markov chain there is at least one stationary distribution.
3. For a countable state Markov chain, every state can be transient.
4. For an aperiodic countable state Markov chain there is at least one stationary distribution.

107. Suppose X follows an exponential distribution with parameter $\lambda > 0$. Fix $a > 0$. Define the random variable Y by

$Y = k$, if $ka \leq X < (k+1)a$,

$k = 0,1,2,\dots$

Which of the following statements are correct?

1. $P(4 < Y < 5) = 0$
2. Y follows an exponential distribution
3. Y follows a geometric distribution
4. Y follows a Poisson distribution

108. Let $\{X_1, \dots, X_n\}$ be a random sample from the probability density function

$f(x;\theta) = \frac{1}{2} e^{-|x-\theta|}; -\infty < x < \infty$ where $\theta \in \mathbb{R}$.

Which of the following statements are correct?

1. The maximum likelihood estimator of θ is $\frac{1}{n}\sum_{i=1}^{n} X_i$.
2. $\sum_{i=1}^{n} X_i$ is a sufficient statistic for θ
3. The maximum likelihood estimator of θ is a function of a sufficient statistic
4. There does not exist a *uniformly most powerful* test for the following testing problem: $H_0: \theta = 0$ vs $H_1: \theta \neq 0$

109. Consider the problem of testing $H_0: \theta = 1$ vs $H_1: \theta = \frac{1}{2}$ where θ is the mean of a Poisson random variable. Let X and Y be a random sample from Poisson (θ) distribution. Consider the following test procedure:

Reject H_0 if either $X = 0$ or ($X = 1$ and $X + Y \leq 2$); otherwise accept H_0.

Which of the following are true?

1. P[type I error] $= e^{-1} + 2e^{-2}$
2. P[type II error] $= 1 - \frac{1}{2}e^{-1} - e^{-\frac{1}{2}}$
3. Size of the test is $e^{-1} + e^{-2}$
4. Power of the test is $\frac{3}{4}e^{-1} + e^{-\frac{1}{2}}$

110. Suppose $\{X_1, \dots, X_n\}$ is a random sample from the distribution with probability density function $f(x)$. Consider the following testing problem using likelihood ratio test (LRT).

H_0: $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ vs

H_1: $f(x) = \frac{1}{2} e^{-|x|}$

Which of the following statements are correct?

1. There does not exist any LRT.
2. The rejection region is a function of $|X_1|, \dots, |X_n|$.
3. The rejection region is a function of $X_1^2, \dots, X_n^2$.
4. The rejection region is of the form $\{\sum_{i=1}^{n} (|X_i| - 1)^2 \geq c\}$.

111. Let $X_1, X_2, \dots, X_7$ be i.i.d. random variables with common continuous distribution function $F(x - \theta_1)$ and let $Y_1, Y_2, \dots, Y_7$ be i.i.d. random variables with common continuous distribution function $F(y - \theta_2)$. Consider the problem of testing

$$H_0: \theta_1 = \theta_2 \text{ vs } H_1: \theta_1 > \theta_2.$$

Let $R_1, R_2, \ldots R_7, R_8, R_9, \ldots, R_{14}$ be the ranks of $X_1, X_2, \ldots, X_7, Y_1, Y_2, \ldots, Y_7$, respectively in the combined sample. Define

$$T_1 = \sum_{i=1}^{7} R_i \text{ and } T_2 = \sum_{j=8}^{14} R_j.$$

Which of the following statements are true?

1. $E(T_1) = E(T_2)$ under H_0
2. $E(T_1) = 52.5$ under H_0
3. T_2 cannot be 27
4. If we use right-tailed test based on T_1, then the observed value $T_1 = 77$ is significant at 5% level of significance.

112. In a football league, the goals scored by home teams over 380 matches have the following frequency distribution.

Number of goals	0	1	2	3	4	5
Frequency	92	121	91	50	19	7

The average goals scored by home teams is 1.49. We want to test

H_0: Goal distribution is Poisson.

Based on observations the value of the χ^2-statistic for goodness of fit is 1.27. Given $\chi^2_{0.05,6} = 1.64$, $\chi^2_{0.05,5} = 1.15$, $\chi^2_{0.95,6} = 12.59$ and $\chi^2_{0.95,5} = 11.07$, which of the following are true?

1. H_0 is not rejected at 5% level of significance.
2. χ^2-statistic has 5 degrees of freedom
3. Under H_0, the maximum likelihood estimate (MLE) of the rate parameter of Poisson is 1.49
4. Under H_0, in a game, the MLE of the probability that home team will score at most one goal is $2.49e^{-1.49}$

113. Consider the model $Y = X\beta + \epsilon$,

where $Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$, $X = \left((x_{ij})\right)_{n\times p}$ and

$$\beta = \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_p \end{bmatrix}, \quad \epsilon = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{bmatrix}.$$

$E(\epsilon) = 0$ and $D(\epsilon) = \sigma^2 I_n$, $p < n$.
Let $\hat{\beta}$ be the solution of $X^T X\beta = X^T Y$.
Which of the following are true?

1. If $C^T\beta$ is estimable then $C^T\hat{\beta}$ is the best linear unbiased estimator (BLUE) of $C^T\beta$
2. All linear parametric functions are estimable if and only if Rank $(X) > p$.
3. If Rank$(X) < p$ then some linear parametric functions are not estimable.
4. $(Y - X\hat{\beta})^T(Y - X\hat{\beta})/(n-p)$ is an unbiased estimator of σ^2.

114. Let X_1 and X_2 be independent random variables each having $N(\mu, \sigma^2)$ distribution, where $\mu \in \mathbb{R}, \sigma^2 > 0$. Let $0 \le \theta < 2\pi$ and $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$. Put $\underline{Y} = (Y_1, Y_2)^t = A\underline{X}$ with $\underline{X} = (X_1, X_2)^t$. Which of the following statements are correct?

1. $\underline{Y} = \underline{X}$ in distribution if and only if $\mu = 0$.
2. $\underline{Y} = \underline{X}$ in distribution if and only if $\mu = 0,\ \theta = 0$.
3. Y_1 and Y_2 are Gaussian.
4. Y_1 and Y_2 may be correlated.

115. Let X_1 and X_2 be two i.i.d. $N_p(0, \Sigma)$ random variables with rank$(\Sigma) = p$. Suppose A is a $p \times p$ symmetric matrix of rank r, and $A^2 = A$. Which of the following statements are correct?

1. $X_1^T A X_1 \sim \chi^2_r$
2. $X_1^T A X_1 + X_2^T A X_2 \sim 2\chi^2_r$
3. $X_1^T A X_2 + X_2^T A X_1 \sim 2\chi^2_r$
4. $X_1^T A X_1 + X_2^T A X_2 \sim \chi^2_{2r}$

116. Consider a finite population of size N. Let T_1 be the sample mean based on a sample of size n under simple random sampling with replacement (SRSWR) scheme. Let T_2 be the sample mean based on a stratified random sample of size n where the samples are drawn from each of 4 strata using

SRSWR scheme under proportional allocation. Then which of the following are sufficient conditions for $\text{Var}(T_1) = \text{Var}(T_2)$ to hold?

1. Strata sizes are same
2. Strata totals are same
3. Strata means are same
4. Strata variances are same

117. Consider a balanced incomplete block design (BIBD) with parameters b, v, r, k, λ where each of the b blocks contains k treatments out of a set of v treatments, each treatment occurs r times in the design and each pair of treatments occurs λ times. A new design is formed by replacing all the treatments in each block by its complementary set. Then which of the following are true for the new design?

1. It is a BIBD
2. Each treatment occurs $(b - r)$ times
3. Each pair of treatments appears in the same block $(b - r + \lambda)$ number of times
4. $bk = vr$

118. Suppose the random variable X has the following probability density function

$$f(x) = \begin{cases} \alpha(x-\mu)^{\alpha-1} e^{-(x-\mu)^\alpha}; & x > \mu \\ 0 & x \le \mu, \end{cases}$$

where $\alpha > 0, -\infty < \mu < \infty$. Which of the following statements are correct? The hazard function of X is

1. an increasing function for all $\alpha > 0$
2. a decreasing function for all $\alpha > 0$
3. an increasing function for some $\alpha > 0$
4. a decreasing function for some $\alpha > 0$

119. Consider the problem:

Maximize $2y_1 + 3y_2 + 5y_3 + 4y_4$

subject to

$y_1 + y_2 \le 1, y_2 + y_3 \le 1, y_3 + y_4 \le 1,$

$y_4 + y_1 \le 1$ and $y_i \ge 0$ for $i = 1,2,3,4.$

Then the optimum value is

1. equal to 8
2. between 8 and 9
3. greater than or equal to 7
4. less than or equal to 7

120. Let (x_1, x_2, x_3, x_4) be an optimal solution to the problem of minimizing

$x_1 + x_2 + x_3 + x_4$

subject to the constraints

$x_1 + x_2 \ge 300$

$x_2 + x_3 \ge 500$

$x_3 + x_4 \ge 400$

$x_4 + x_1 \ge 200$

$x_1 \ge 0, x_2 \ge 0, \quad x_3 \ge 0, x_4 \ge 0.$

Which of the following are **not** possible values for any x_i?

1. 300
2. 400
3. 500
4. 600

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JOINT CSIR-UGC-JRF/NET JUNE 2017 ANSWER KEY BOOKLET A

MATHEMATICAL SCIENCE ENGLISH AND BILINGUAL

Q. No.	Key	31	4	62	2,3	93	1,3
1	4	32	3	63	3	94	3,4
2	1	33	3	64	1,2,3	95	1,3
3	1	34	2	65	3,4	96	1,3,4
4	4	35	1	66	3,4	97	4
5	4	36	3	67	2	98	2
6	3	37	1	68	3,4	99	3
7	4	38	1	69	1,3,4	100	1
8	4	39	4	70	2,4	101	1,3
9	3	40	3	71	2,3	102	1,2
10	4	41	2	72	2,4	103	2,4
11	1	42	1	73	1,3,4	104	2,3,4
12	1	43	1	74	1,2,4	105	2,4
13	2	44	1	75	1,3	106	2,3
14	1	45	3	76	1,4	107	1,3
15	1	46	2	77	1,4	108	3,4
16	2	47	2	78	1	109	1,4
17	4	48	2	79	1,2	110	2,3,4
18	3	49	3	80	3,4	111	1,2,3,4
19	3	50	4	81	2,4	112	1,2,3,4
20	2	51	1	82	2,4	113	1,3,4
21	1	52	1	83	1,2,4	114	1,3
22	4	53	2	84	2,4	115	1,4
23	3	54	2	85	1,4	116	3
24	3	55	4	86	1,4	117	1,2,4
25	3	56	4	87	2,4	118	3,4
26	1	57	4	88	1,4	119	3,4
27	2	58	4	89	2,3,4	120	3,4
28	1	59	1	90	1,3		
29	2	60	3	91	3		
30	2	61	2,3,4	92	2,3		