Sr. No. **17912**

E

SUBJECT CODE BOOKLET CODE

4 **B**

2017 (II)
MATHEMATICAL SCIENCES
TEST BOOKLET

Time : 3:00 Hours *Maximum Marks: 200*

INSTRUCTIONS

1. **You have opted for English as medium of Question Paper.** This Test Booklet contains one hundred and twenty (20 Part'A'+40 Part 'B' +60 Part 'C') Multiple Choice Questions (MCQs). You are required to answer a maximum of 15, 25 and 20 questions from part 'A' 'B' and 'C' respectively. If more than required number of questions are answered, only first 15, 25 and 20 questions in Parts 'A' 'B' and 'C' respectively, will be taken up for evaluation.
2. **OMR** answer sheet has been provided separately. Before you start filling up your particulars, please ensure that the booklet contains requisite number of pages and that these are not torn or mutilated. If it is so, you may request the Invigilator to change the booklet of the same code. Likewise, check the **OMR** answer sheet also. Sheets for rough work have been appended to the test booklet.
3. Write your Roll No., Name and Serial Number of this Test Booklet on the **OMR** Answer sheet in the space provided. Also put your signatures in the space earmarked.
4. **You must darken the appropriate circles with a black ball pen related to Roll Number, Subject Code, Booklet Code and Centre Code on the OMR answer sheet. It is the sole responsibility of the candidate to meticulously follow the instructions given on the OMR Answer Sheet, failing which, the computer shall not be able to decipher the correct details which may ultimately result in loss, including rejection of the OMR answer sheet.**
5. Each question in Part 'A' carries **2** marks, Part 'B' **3** marks and Part 'C' **4.75** marks respectively. There will be negative marking @ 0.5marks in Part 'A' and @ 0.75 marks in Part 'B' for each wrong answer and no negative marking for Part 'C'.
6. Below each question in Part 'A' and 'B', four alternatives or responses are given. Only one of these alternatives is the "correct" option to the question. You have to find, for each question, the correct or the best answer. In Part 'C' each question may have **'ONE'** or **'MORE'** correct options. Credit in a question shall be given only on identification of **'ALL'** the correct options in Part 'C'.
7. Candidates found copying or resorting to any unfair means are liable to be disqualified from this and future examinations.
8. Candidate should not write anything anywhere except on **OMR** answer sheet or sheets for rough work.
9. Use of calculator is not permitted.
10. **After the test is over, at the perforation point, tear the OMR answer sheet, hand over the original OMR answer sheet to the invigilator and retain the carbonless copy for your record.**
11. Candidates who sit for the entire duration of the exam will only be permitted to carry their Test booklet.

Roll No. 411127....................

Name DIVYA NAIR..........

I have verified all the information filled in by the candidate.

....

Signature of the Invigilator

PART A

1. A new tyre can be used for at most 90 km. What is the maximum distance (in km) that can be covered by a three wheeled vehicle carrying one spare wheel, all four tyres being new?

1. 180   2. 90
3. 120   4. 270

2. A 2 m long ladder is to reach a wall of height 1.75 m. The largest possible horizontal distance of the ladder from the wall could be

1. slightly less than 1 m
2. slightly more than 1 m
3. 1 m
4. 1.2 m

3. For which one of the following statements is the converse NOT true?

1. If a patient dies even with excellent medical care, he likely had terminal illness.
2. If a person gets employed, he has good qualifications.
3. If an integer is even, it is divisible by two.
4. If an integer is odd, it is not divisible by two.

4. What is the maximum number of cylindrical pencils of 0.5 cm diameter that can be stood in a square shaped stand of 5 cm × 5 cm inner cross section?

1. 99   2. 121
3. 100   4. 105

5. If a plant with green leaves is kept in a dark room with only green light ON, which one of the following would we observe?

1. The plant appears brighter than the surroundings
2. The plant appears darker than the surroundings
3. We cannot distinguish the plant from the surroundings
4. It will have above normal photosynthetic activity

6. Four small squares of side x are cut out of a square of side 12 cm to make a tray by folding the edges. What is the value of x so that the tray has the maximum volume?

1. 6 cm   2. 2 cm
3. 3 cm   4. 4 cm

7. The smallest square floor which can be completely paved with tiles of size 8×6, without breaking any tile, needs n tiles. Find n.

1. 56   2. 12
3. 24   4. 48

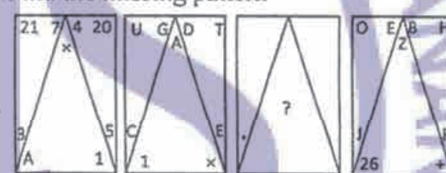
8. The sum of two numbers is equal to sum of square of 11 and cube of 9. The larger number is $(5)^2$ less than square of 25. What is the value of the sum of twice of 24 percent of the smaller number and half of the larger number?

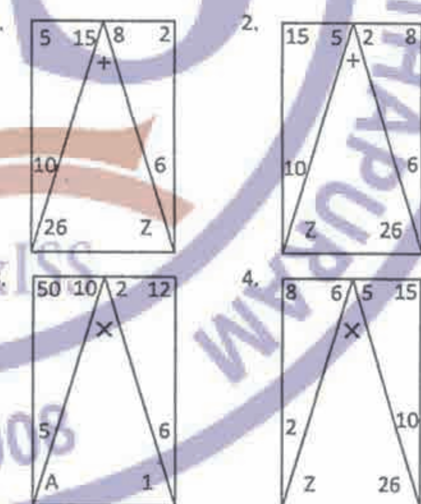
1. 415   2. 400
3. 410   4. 420

9. There are small and large bacteria of the same species. If S is surface area and V is volume, then which of the following is correct?

1. $S_{\text{small}} > S_{\text{large}}$
2. $V_{\text{small}} > V_{\text{large}}$
3. $(S/V)_{\text{small}} > (S/V)_{\text{large}}$
4. $(S/V)_{\text{small}} < (S/V)_{\text{large}}$

10. Find the missing pattern





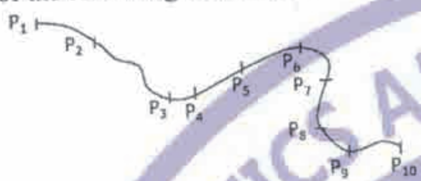
11. Two runners A and B start running from diametrically opposite points on a circular track in the same direction. If A runs at a constant speed of 8 km/h and B at a constant speed of 6

CENTRE FOR MATHEMATICS & STATISTICS TRIVANDRUM
Phone: 8113887329, 8075791772
Email: cmscsirnet@gmail.com

km/h and A catches up with B in 30 minutes, what is the length of the track?

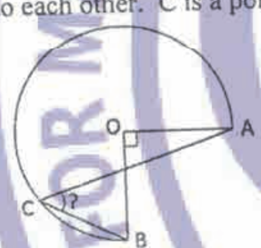
1. 1 km
2. 4 km
3. 3 km
4. 2 km

12. A path between points P_1 and P_{10} on a level ground is shown, and' positions of a moving object at 1 second intervals are marked. Which of the following statements is correct?



1. The motion is uniform
2. The speed between P_3 and P_4 is greater than that between P_5 and P_6
3. The speed from P_1 to P_2 increases because of downward slope
4. The section P_3 to P_4 is covered at the slowest speed

13. Three-quarters of a circle is shown in the figure; OA and OB are two radii perpendicular to each other. C is a point on the circle.



What is angle ACB?

1. Cannot be determined
2. 30°
3. 60°
4. 45°

14. A plate of $5m \times 2m$ size with uniform thickness, weighing 20 kg, is perforated with 1000 holes of $5cm \times 2cm$ size. What is the weight of the plate (in kg) after perforation?

1. 10
2. 2
3. 19.8
4. 18

15. What is the volume of soil in an open pit of size 2 m × 2 m × 10 cm?

1. 40 m^3
2. 0.4 m^3
3. 0 m^3
4. 4.0 m^3

16. For which values of A and B is $\sin A = \cot B$?

1. $A = B = 0$
2. $A = B = \frac{\pi}{2}$
3. $A = 0,\ B = \frac{\pi}{2}$
4. $A = \frac{\pi}{2},\ B = 0$

17. A rectangular flask of length 11 cm, width 8 cm and height 20 cm has water filled up to height 5 cm. If 21 spherical marbles of radius 1 cm each are dropped in the flask, what would be the rise in water level?

1. 8.8 cm
2. 10 cm
3. 1 cm
4. 0 cm

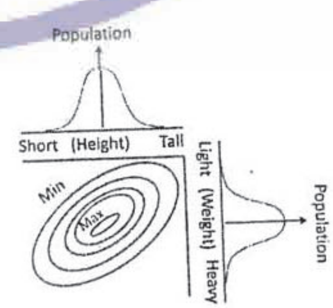
18. A boy holds one end of a rope of length l and the other end is fixed to a thin pole of radius r ($r \ll l$). Keeping the rope taut, the boy goes around the pole causing the rope to get wound around the pole. Each round takes 10 s. What is the speed (in units of s^{-1}) with which the boy approaches the pole?

1. $\frac{\pi r}{5}$
2. $\frac{\pi l}{5}$
3. $20\pi(r+l)$
4. $\frac{2\pi(l-r)}{5}$

19. A person purchases two chains from a jeweller, one weighing 18 g made of 22 carat gold and another weighing 22 g made of 18 carat gold. Which one of the following statements is correct?

1. 22 carat chain contains $\frac{2}{11}$ times more gold than 18 carat chain
2. 22 carat chain contains $\frac{1}{11}$ times more gold than 18 carat chain
3. Both chains contain the same quantity of gold
4. 18 carat chain contains $\frac{2}{11}$ times more gold than 22 carat chain

20. Contours in the bivariate (weight, height) graph connect regions of approximately equal populations. Which of the following interpretations is correct?



CENTRE FOR MATHEMATICS & STATISTICS TRIVANDRUM
Phone: 8113887329, 8075791772
Email: cmscsirnet@gmail.com

1. There is no correlation between height and weight of the population
2. Heavier individuals are likely to be taller than lighter individuals
3. Taller and lighter individuals are more in number than taller and heavier individuals
4. There are no individuals of medium weight and medium height

PART B

UNIT-1

21. Which of the following is necessarily true for a function $f: X \to Y$?
    1. if f is injective, then there exists $g: Y \to X$ such that $f(g(y)) = y$ for all $y \in Y$
    2. if f is surjective, then there exists $g: Y \to X$ such that $f(g(y)) = y$ for all $y \in Y$
    3. if f is injective and Y is countable then X is finite
    4. if f is surjective and X is uncountable then Y is countably infinite

22. Let $S = \{f: \mathbb{R} \to \mathbb{R} \mid \exists\, \epsilon > 0$ such that $\forall \delta > 0, |x - y| < \delta \Rightarrow |f(x) - f(y)| < \epsilon\}$. Then
    1. $S = \{f: \mathbb{R} \to \mathbb{R} \mid f$ is continuous$\}$
    2. $S = \{f: \mathbb{R} \to \mathbb{R} \mid f$ is uniformly continuous$\}$
    3. $S = \{f: \mathbb{R} \to \mathbb{R} \mid f$ is bounded$\}$
    4. $S = \{f: \mathbb{R} \to \mathbb{R} \mid f$ is constant$\}$

23. Let $f: (0, \infty) \to \mathbb{R}$ be uniformly continuous. Then
    1. $\lim_{x \to 0+} f(x)$ and $\lim_{x \to \infty} f(x)$ exist
    2. $\lim_{x \to 0+} f(x)$ exists but $\lim_{x \to \infty} f(x)$ need not exist
    3. $\lim_{x \to 0+} f(x)$ need not exist but $\lim_{x \to \infty} f(x)$ exists
    4. neither $\lim_{x \to 0+} f(x)$ nor $\lim_{x \to \infty} f(x)$ need exist

24. Let D be a subset of the real line. Consider the assertion: "Every infinite sequence in D has a subsequence which converges in D". This assertion is true if
    1. $D = [0, \infty)$
    2. $D = [0,1] \cup [3,4]$
    3. $D = [-1,1) \cup (1,2]$
    4. $D = (-1, 1]$

25. Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers satisfying $a_1 \geq 1$ and $a_{n+1} \geq a_n + 1$ for all $n \geq 1$. Then which of the following is necessarily true?
    1. The series $\sum_{n=1}^{\infty} \frac{1}{a_n^2}$ diverges
    2. The sequence $\{a_n\}_{n\geq 1}$ is bounded
    3. The series $\sum_{n=1}^{\infty} \frac{1}{a_n^2}$ converges
    4. The series $\sum_{n=1}^{\infty} \frac{1}{a_n}$ converges

26. Let $\mathbb{Z}$ denote the set of integers and $\mathbb{Z}_{\geq 0}$ denote the set $\{0, 1, 2, 3, \ldots\}$. Consider the map $f: \mathbb{Z}_{\geq 0} \times \mathbb{Z} \to \mathbb{Z}$ given by $f(m, n) = 2^m \cdot (2n + 1)$. Then the map f is
    1. onto (surjective) but not one–one (injective)
    2. one–one (injective) but not onto (surjective)
    3. both one–one and onto
    4. neither one–one nor onto

27. Let $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \in M_2(\mathbb{R})$ and $\phi: \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ be the bilinear map defined by $\phi(v, w) = v^T A w$. Choose the correct statement from below:
    1. $\phi(v, w) = \phi(w, v)$ for all $v, w \in \mathbb{R}^2$
    2. there exists nonzero $v \in \mathbb{R}^2$ such that $\phi(v, w) = 0$ for all $w \in \mathbb{R}^2$
    3. there exists a 2×2 symmetric matrix B such that $\phi(v, v) = v^T B v$ for all $v \in \mathbb{R}^2$
    4. the map $\psi: \mathbb{R}^4 \to \mathbb{R}$ defined by $\psi\left(\begin{bmatrix} v_1 \\ v_2 \\ w_1 \\ w_2 \end{bmatrix}\right) = \phi\left(\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}\right)$ is linear

28. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 2 & 3 & \alpha \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 3 \\ \beta \end{bmatrix}$. Then the system $AX = b$ over the real numbers has
    1. no solution whenever $\beta \neq 7$
    2. an infinite number of solutions whenever $\alpha \neq 2$
    3. an infinite number of solutions if $\alpha = 2$ and $\beta \neq 7$
    4. a unique solution if $\alpha \neq 2$

29. Let $A = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$. Then the smallest positive integer n such that $A^n = I$ is
    1. 1
    2. 2
    3. 4
    4. 6

30. Let A be a real symmetric matrix and $B = I + iA$, where $i^2 = -1$. Then
    1. B is invertible if and only if A is invertible
    2. all eigenvalues of B are necessarily real

CENTRE FOR MATHEMATICS & STATISTICS TRIVANDRUM
Phone: 8113887329, 8075791772
Email: cmscsirnet@gmail.com

3. $B - I$ is necessarily invertible
4. B is necessarily invertible

31. Let $S = \{x \in [-1,4] \mid \sin(x) > 0\}$. Which of the following is true?
1. $\inf(S) < 0$
2. $\sup(S)$ does not exist
3. $\sup(S) = \pi$
4. $\inf(S) = \pi/2$

32. Let k be a positive integer and let
$S_k = \{x \in [0,1] \mid$ a decimal expansion of x has a prime digit at its k^{th} place$\}$.
Then the Lebesgue measure of S_k is
1. 0
2. 4/10
3. $(4/10)^k$
4. 1

Unit-2

33. Let $\mathbb{D}$ be the open unit disc in the complex plane and $U = \mathbb{D} \setminus \left\{-\frac{1}{2}, \frac{1}{2}\right\}$. Also, let
$H_1 = \{f: \mathbb{D} \to \mathbb{C} \mid f$ is holomorphic and bounded$\}$ and
$H_2 = \{f: U \to \mathbb{C} \mid f$ is holomorphic and bounded$\}$.
Then the map $r: H_1 \to H_2$ given by $r(f) = f|_U$, the restriction of f to U, is
1. injective but not surjective
2. surjective but not injective
3. injective and surjective
4. neither injective nor surjective

34. Let C be the circle of radius 2 with centre at the origin in the complex plane, oriented in the anti-clockwise direction. Then the integral $\oint_C \frac{dz}{(z-1)^2}$ is equal to
1. $\frac{1}{2\pi i}$
2. $2\pi i$
3. 1
4. 0

35. Let f be a holomorphic function in the open unit disc such that $\lim_{z\to 1} f(z)$ does not exist. Let $\sum_{n=0}^{\infty} a_n z^n$ be the Taylor series of f about $z = 0$ and let R be its radius of convergence. Then
1. $R = 0$
2. $0 < R < 1$
3. $R = 1$
4. $R > 1$

36. The function $f: \mathbb{C} \to \mathbb{C}$ defined by $f(z) = e^z + e^{-z}$ has
1. finitely many zeros
2. no zeros
3. only real zeros
4. has infinitely many zeros

37. Let A be a connected open subset of $\mathbb{R}^2$. The number of continuous surjective functions from $\bar{A}$ (the closure of A in $\mathbb{R}^2$) to $\mathbb{Q}$ is:
1. 1
2. 0
3. 2
4. not finite

38. Let R be a subring of $\mathbb{Q}$ containing 1. Then which of the following is necessarily true?
1. R is a principal ideal domain (PID)
2. R contains infinitely many prime ideals
3. R contains a prime ideal which is not a maximal ideal
4. for every maximal ideal m in R, the residue field R/m is finite

39. The group S_3 of permutations of $\{1,2,3\}$ acts on the three dimensional vector space over the finite field $\mathbb{F}_3$ of three elements, by permuting the vectors in basis $\{e_1, e_2, e_3\}$ by $\sigma \cdot e_i = e_{\sigma(i)}$, for all $\sigma \in S_3$. The cardinality of the set of vectors fixed under the above action is
1. 0
2. 3
3. 9
4. 27

40. Let $f: \mathbb{Z} \to (\mathbb{Z}/4\mathbb{Z}) \times (\mathbb{Z}/6\mathbb{Z})$ be the function $f(n) = (n \bmod 4,\ n \bmod 6)$. Then
1. $(0 \bmod 4, 3 \bmod 6)$ is in the image of f
2. $(a \bmod 4,\ b \bmod 6)$ is in the image of f, for all even integers a and b
3. image of f has exactly 6 elements
4. kernel of $f = 24\mathbb{Z}$

UNIT 3

41. Let $u(x,t)$ be the solution of the initial value problem
$$u_{tt} - u_{xx} = 0$$
$$u(x,0) = x^3$$
$$u_t(x,0) = \sin x.$$
Then $u(\pi,\pi)$ is
1. $4\pi^3$
2. π^3
3. 0
4. 4

42. The set of real numbers λ for which the boundary value problem
$$\frac{d^2y}{dx^2} + \lambda y = 0, \quad y(0) = 0, \quad y(\pi) = 0$$
has nontrivial solutions is
1. $(-\infty, 0)$
2. $\{\sqrt{n} \mid n$ is a positive integer$\}$

CENTRE FOR MATHEMATICS & STATISTICS TRIVANDRUM
Phone: 8113887329, 8075791772
Email: cmscsirnet@gmail.com

3. $\{n^2 \mid n \text{ is a positive integer}\}$
4. $\mathbb{R}$

43. Let D denote the unit disc given by $\{(x,y) \mid x^2+y^2 \le 1\}$ and let D^c be its complement in the plane. The partial differential equation
$(x^2-1)\frac{\partial^2 u}{\partial x^2} + 2y\frac{\partial^2 u}{\partial x \partial y} - \frac{\partial^2 u}{\partial y^2} = 0$ is
1. parabolic for all $(x,y) \in D^c$
2. hyperbolic for all $(x,y) \in D$
3. hyperbolic for all $(x,y) \in D^c$
4. parabolic for all $(x,y) \in D$

44. Consider the differential equation
$(x-1)y'' + xy' + \frac{1}{x}y = 0.$
Then
1. $x = 1$ is the only singular point
2. $x = 0$ is the only singular point
3. both $x = 0$ and $x = 1$ are singular points
4. neither $x = 0$ nor $x = 1$ are singular points

45. Let $I(m)$ denote the moment of inertia of a regular solid tetrahedron about an axis m passing through its centre of gravity. Which of the following is true?
1. if the axis ℓ passes through a vertex and the axis ℓ' does not pass through a vertex then $I(\ell) > I(\ell')$
2. if the axis ℓ passes through the mid-point of an edge and ℓ' is any other axis then $I(\ell) > I(\ell')$
3. $I(\ell)$ is the same for all axes ℓ
4. if the axis ℓ passes through a vertex and the axis ℓ' does not pass through a vertex then $I(\ell) < I(\ell')$

46. Let $u(x,t)$ be a solution of the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ in a rectangle $[0,\pi] \times [0,T]$ subject to the boundary conditions $u(0,t) = u(\pi,t) = 0$, $0 \le t \le T$ and the initial condition $u(x,0) = \varphi(x)$, $0 \le x \le \pi$. If $f(x) = u(x,T)$, then which of the following is true for a suitable kernel $k(x,y)$?

1. $\int_0^{\pi} k(x,y)\,\varphi(y)dy = f(x), \quad 0 \le x \le \pi$
2. $\varphi(x) + \int_0^{\pi} k(x,y)\varphi(y)dy = f(x), \quad 0 \le x \le \pi$
3. $\int_0^{x} k(x,y)\varphi(y)dy = f(x), \quad 0 \le x \le \pi$
4. $\varphi(x) + \int_0^{x} k(x,y)\varphi(y)dy = f(x), \quad 0 \le x \le \pi$

47. Let $X = \{u \in C^1[0,1] \mid u(0) = u(1) = 0\}$ and define $J: X \to \mathbb{R}$ by
$J(u) = \int_0^1 e^{-u'(x)^2}dx.$
Then
1. J does not attain its infimum
2. J attains its infimum at a unique $u \in X$
3. J attains its infimum at exactly two elements $u \in X$
4. J attains its infimum at infinitely many $u \in X$

48. The iterative method $x_{n+1} = g(x_n)$ for the solution of $x^2 - x - 2 = 0$ converges quadratically in a neighbourhood of the root $x = 2$ if $g(x)$ equals
1. $x^2 - 2$
2. $(x-2)^2 - 6$
3. $1 + \frac{2}{x}$
4. $\frac{x^2+2}{2x-1}$

UNIT 4

49. Let X be a random sample of size 1 from a Cauchy distribution with probability density function
$f_\theta(x) = \frac{1}{\pi}\left(\frac{1}{1+(x-\theta)^2}\right), -\infty < x < \infty,$
where $\theta \in (-\infty, \infty)$. For testing $H_0: \theta = -1$ against $H_1: \theta = 0$, the following test is suggested.
Reject H_0 if $\frac{X}{\sqrt{1+X^2}} > C$, otherwise do not reject H_0.
What is the value of C so that the power of the test is 0.5?
1. $\frac{\pi}{4}$
2. 0
3. $\tan^{-1}\left(\frac{1}{2}\right)$
4. a solution of $\tan^{-1}\sqrt{\frac{C}{1-C}} = \frac{\pi}{3}$

50. Let X_1 and X_2 be a random sample of size two from a distribution with probability density function
$f_\theta(x) = \theta\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2} + (1-\theta)\frac{1}{2}e^{-|x|},$
$-\infty < x < \infty,$

CENTRE FOR MATHEMATICS & STATISTICS TRIVANDRUM
Phone: 8113887329, 8075791772
Email: cmscsirnet@gmail.com

where $\theta \in \left\{0, \frac{1}{2}, 1\right\}$. If the observed values of X_1 and X_2 are 0 and 2, respectively, then the maximum likelihood estimate of θ is

1. 0
2. $\frac{1}{2}$
3. 1
4. not unique

51. X, Y are independent exponential random variables with means 4 and 5, respectively. Which of the following statements is true?
    1. $X + Y$ is exponential with mean 9
    2. XY is exponential with mean 20
    3. max (X, Y) is exponential
    4. min (X, Y) is exponential

52. Consider a Markov chain $\{X_n \mid n \geq 0\}$ with state space $\{1, 2, 3\}$ and transition matrix

$$P = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}.$$ Then $P(X_3 = 1 \mid X_0 = 1)$ equals

1. 0
2. $\frac{1}{4}$
3. $\frac{1}{2}$
4. $\frac{1}{8}$

53. Let $\psi(t) = e^{-|t|-\frac{t^2}{2}}$ and $\varphi(t) = \frac{e^{-|t|}+e^{-\frac{t^2}{2}}}{2}$. Which of the following is true?
    1. ψ is a characteristic function but φ is not
    2. φ is a characteristic function but ψ is not
    3. both ψ and φ are characteristic functions
    4. neither ψ nor φ is a characteristic function

54. There are five empty boxes. Balls are placed independently one after another in randomly selected boxes. The probability that the fourth ball is the first to be placed in an occupied box equals
    1. $\frac{4}{5}\left(\frac{3}{5}\right)^2$
    2. $\left(\frac{3}{5}\right)^3$
    3. $\left(\frac{3}{5}\right)^2$
    4. $\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$

55. A parallel system consists of n identical components. The lifetimes of the components are independent identically distributed uniform random variables with mean 30 hours and range 60 hours. If the expected lifetime of the system is 50 hours, then the value of n is
    1. 3
    2. 4
    3. 5
    4. 6

56. Let X and Y be independent exponential random variables. If $E[X] = 1$ and $E[Y] = \frac{1}{2}$ then $P(X > 2Y \mid X > Y)$ is
    1. $\frac{1}{2}$
    2. $\frac{1}{3}$
    3. $\frac{2}{3}$
    4. $\frac{3}{4}$

57. Suppose we draw a random sample of size n from a population of size N, where $1 < n < N$, using simple random sampling without replacement scheme. Let P be the population proportion of units possessing a particular attribute and p be the corresponding sample proportion. Which of the following is an unbiased estimator for $P(1-P)$?
    1. $p(1-p)$
    2. $\frac{N-n}{N-1}p(1-p)$
    3. $\frac{n(N-1)}{N(n-1)}p(1-p)$
    4. $\frac{N(n-1)}{n(N-1)}p(1-p)$

58. Suppose $X_1 \sim N_{p_1}(0, \Sigma_1)$, $X_2 \sim N_{p_2}(0, \Sigma_2)$, where X_1 and X_2 are independently distributed. If $p_1 > p_2$ and Σ_1, Σ_2 are positive definite then which of the following statements is necessarily true?
    1. $X_1^T\Sigma_1 X_1 + X_2^T\Sigma_2 X_2 \sim \chi^2_{p_1+p_2}$
    2. $X_1^T\Sigma_1^{-1} X_1 + X_2^T\Sigma_2^{-1} X_2 \sim \chi^2_{p_1+p_2}$
    3. $X_1^T\Sigma_1^{-1} X_1 - X_2^T\Sigma_2^{-1} X_2 \sim \chi^2_{p_1-p_2}$
    4. $\frac{p_1 X_1^T\Sigma_1^{-1} X_1}{p_2 X_2^T\Sigma_2^{-1} X_2} \sim F_{p_1,p_2}$

59. Consider the following regression problem $Y_i = \alpha + \beta i + \epsilon_i;\ i = 1, \ldots, n.$ Here $\epsilon_i, i = 1, 2, \ldots, n$, are i.i.d $N(0,1)$ random variables. It is assumed that $\alpha \neq 0$ and β is known. If $\hat{\alpha}_n$ is the MLE of α, which of the following statements is true?
    1. $\lim_{n\to\infty} E(\hat{\alpha}_n) \neq \alpha$
    2. $\lim_{n\to\infty} E(\hat{\alpha}_n) = 0$
    3. $\lim_{n\to\infty} \text{Var}(\hat{\alpha}_n) = \infty$
    4. $\lim_{n\to\infty} \text{Var}(\hat{\alpha}_n) = 0$

60. Let $X_1, X_2, \ldots$ be a random sample from uniform $(0, 3\theta), \theta > 0$. Define $T_n = \frac{1}{3}\max\{X_1, X_2, \ldots, X_n\}$. Which of the following is NOT true?

1. T_n is consistent for θ
2. T_n is unbiased for θ
3. T_n is a sufficient statistic
4. T_n is complete

PART C

UNIT-1

61. Evaluate $\lim_{n\to\infty} \sum_{k=0}^{n} \frac{n}{k^2+n^2}$.
1. $\frac{\pi}{2}$
2. π
3. $\frac{\pi}{8}$
4. $\frac{\pi}{4}$

62. Consider the set of rational numbers $\mathbb{Q}$ as a subspace of $\mathbb{R}$ with the usual metric. Suppose a and b are irrational numbers with $a < b$ and let $K = [a, b] \cap \mathbb{Q}$. Then
1. K is a bounded subset of $\mathbb{Q}$
2. K is a closed subset of $\mathbb{Q}$
3. K is a compact subset of $\mathbb{Q}$
4. K is an open subset of $\mathbb{Q}$

63. Let $f: \mathbb{R} \to \mathbb{R}$ be a function satisfying $f(x+y) = f(x)f(y), \forall x, y \in \mathbb{R}$ and $\lim_{x\to 0} f(x) = 1$. Which of the following are necessarily true?
1. f is stricly increasing
2. f is either constant or bounded
3. $f(rx) = f(x)^r$ for every rational $r \in \mathbb{Q}$
4. $f(x) \geq 0, \quad \forall x \in \mathbb{R}$

64. Let $\mathbb{R}$ denote the set of real numbers and $\mathbb{Q}$ the set of all rational numbers. For $0 \leq \epsilon \leq \frac{1}{2}$, let A_ϵ be the open interval $(0, 1-\epsilon)$. Which of the following are true?
1. $sup_{0<\epsilon<\frac{1}{2}}\, sup(A_\epsilon) < 1$
2. $0 < \epsilon_1 < \epsilon_2 < \frac{1}{2} \Rightarrow inf(A_{\epsilon_1}) < inf(A_{\epsilon_2})$
3. $0 < \epsilon_1 < \epsilon_2 < \frac{1}{2} \Rightarrow sup(A_{\epsilon_1}) > sup(A_{\epsilon_2})$
4. $\sup(A_\epsilon \cap \mathbb{Q}) = \sup(A_\epsilon \cap (\mathbb{R}\backslash\mathbb{Q}))$

65. Let a_{mn}, $m \geq 1$, $n \geq 1$ be a double array of real numbers. Define

$$P = \liminf_{n\to\infty} \liminf_{m\to\infty} a_{mn}, \quad Q = \liminf_{n\to\infty} \limsup_{m\to\infty} a_{mn}$$

$$R = \limsup_{n\to\infty} \liminf_{m\to\infty} a_{mn}, \quad S = \limsup_{n\to\infty} \limsup_{m\to\infty} a_{mn}$$

Which of the following statements are necessarily true?
1. $P \leq Q$
2. $Q \leq R$
3. $R \leq S$
4. $P \leq S$

66. Which of the following are convergent?
1. $\sum_{n=1}^{\infty} n^2 2^{-n}$
2. $\sum_{n=1}^{\infty} n^{-2} 2^n$
3. $\sum_{n=2}^{\infty} \frac{1}{n \log n}$
4. $\sum_{n=1}^{\infty} \frac{1}{n \log(1 + 1/n)}$

67. Let A be an $m \times n$ matrix with rank r. If the linear system $AX = b$ has a solution for each $b \in \mathbb{R}^m$, then
1. $m = r$
2. the column space of A is a proper subspace of $\mathbb{R}^m$
3. the null space of A is a non-trivial subspace of $\mathbb{R}^n$ whenever $m = n$
4. $m \geq n$ implies $m = n$

68. Let $\ell^2 = \{x = (x_n)_{n\geq 1} \mid x_n \in \mathbb{R}, \sum_{n=1}^{\infty} x_n^2 < \infty\}$ be the Hilbert space of square summable sequences and let e_k denote the k^{th} co-ordinate vector (with 1 in k^{th} place, 0 elsewhere). Which of the following subspaces is NOT dense in ℓ^2?
1. $\text{span}\{e_1 - e_2,\ e_2 - e_3, e_3 - e_4, \ldots\}$
2. $\text{span}\{2e_1 - e_2,\ 2e_2 - e_3,\ 2e_3 - e_4, \ldots\}$
3. $\text{span}\{e_1 - 2e_2,\ e_2 - 2e_3,\ e_3 - 2e_4, \ldots\}$
4. $\text{span}\{e_2, e_3, e_4, \ldots\}$

69. Consider $X = \left\{\left(x, \sin\frac{1}{x}\right) \middle| 0 < x \leq 1\right\} \cup \{(0, y) | -1 \leq y \leq 1\}$ as a subspace of $\mathbb{R}^2$ and $Y = [0, 1)$ as a subspace of $\mathbb{R}$. Then
1. X is connected
2. X is compact
3. $X \times Y$ (in product topology) is connected
4. $X \times Y$ (in product topology) is compact

70. Let $f: \mathbb{R}^n \to \mathbb{R}^n$ be a continuously differentiable map satisfying $\|f(x) - f(y)\| \geq \|x - y\|$, for all $x, y \in \mathbb{R}^n$. Then
1. f is onto
2. $f(\mathbb{R}^n)$ is a closed subset of $\mathbb{R}^n$
3. $f(\mathbb{R}^n)$ is an open subset of $\mathbb{R}^n$
4. $f(0) = 0$

71. Let $f: \mathbb{R}^4 \to \mathbb{R}$ be defined by $f(x) = x^t A x$, where A is a 4×4 matrix with real entries and x^t denotes the transpose of x. The gradient of f at a point x_0 necessarily is
1. $2Ax_0$
2. $Ax_0 + A^t x_0$
3. $2A^t x_0$
4. Ax_0

72. Let $f(x,y) = \frac{1-\cos(x+y)}{x^2+y^2}$ if $(x,y) \neq (0,0)$
$f(0,0) = \frac{1}{2}$
and $g(x,y) = \frac{1-\cos(x+y)}{(x+y)^2}$ if $x + y \neq 0$
$g(x,y) = \frac{1}{2}$ if $x + y = 0$
Then
1. f is continuous at $(0,0)$
2. f is continuous everywhere except at $(0,0)$
3. g is continuous at $(0,0)$
4. g is continuous everywhere

73. Let A be an $m \times n$ matrix of rank m with $n > m$. If for some non-zero real number α, we have $x^t A A^t x = \alpha\, x^t x$, for all $x \in \mathbb{R}^m$ then $A^t A$ has
1. exactly two distinct eigenvalues
2. 0 as an eigenvalue with multiplicity $n - m$
3. α as a non-zero eigenvalue
4. exactly two non-zero distinct eigenvalues

74. For every 4×4 real symmetric non-singular matrix A, there exists a positive integer p such that
1. $pI + A$ is positive definite
2. A^p is positive definite
3. A^{-p} is positive definite
4. $\exp(pA) - I$ is positive definite

75. Let V be the vector space over $\mathbb{C}$ of all polynomials in a variable X of degree at most 3. Let $D: V \to V$ be the linear operator given by differentiation with respect to X. Let A be the matrix of D with respect to some basis for V. Which of the following are true?
1. A is a nilpotent matrix
2. A is a diagonalizable matrix
3. the rank of A is 2
4. the Jordan canonical form of A is
$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

76. Let A be a 3×3 matrix with real entries. Identify the correct statements.
1. A is necessarily diagonalizable over $\mathbb{R}$
2. if A has distinct real eigenvalues then it is diagonalizable over $\mathbb{R}$
3. if A has distinct eigenvalues then it is diagonalizable over $\mathbb{C}$
4. if all eigenvalues of A are non-zero then it is diagonalizable over $\mathbb{C}$

77. Let $M = \{A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in \mathbb{Z}$ and the eigenvalues of A are in $\mathbb{Q}\}$. Then
1. M is empty
2. $M = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in \mathbb{Z}\}$
3. if $A \in M$ then the eigenvalues of A are in $\mathbb{Z}$
4. if $A, B \in M$ are such that $AB = I$ then $\det A \in \{+1, -1\}$.

78. Let $f: [-1, 1] \to \mathbb{R}$ be a function given by
$$f(x) = \begin{cases} x^2 \cos\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$
Then
1. f is of bounded variation on $[-1, 1]$
2. f' is of bounded variation on $[-1, 1]$
3. $|f'(x)| \leq 1 \quad \forall x \in [-1, 1]$
4. $|f'(x)| \leq 3 \quad \forall x \in [-1, 1]$

UNIT-2

79. Let G be a finite abelian group and $a, b \in G$ with $\text{order}(a) = m$, $\text{order}(b) = n$. Which of the following are necessarily true?
1. $\text{order}(ab) = mn$
2. $\text{order}(ab) = \text{lcm}(m, n)$
3. there is an element of G whose order is $\text{lcm}(m, n)$
4. $\text{order}(ab) = \text{gcd}(m, n)$

80. For a set X, let $\mathcal{P}(X)$ be the set of all subsets of X and let $\Omega(X)$ be the set of all functions $f: X \to \{0, 1\}$. Then

CENTRE FOR MATHEMATICS & STATISTICS TRIVANDRUM
Phone: 8113887329, 8075791772
Email: cmscsirnet@gmail.com

1. if X is finite then $\mathcal{P}(X)$ is finite
2. if X and Y are finite sets and if there is a 1-1 correspondence between $\mathcal{P}(X)$ and $\mathcal{P}(Y)$, then there is a 1-1 correspondence between X and Y
3. there is no 1-1 correspondence between X and $\mathcal{P}(X)$
4. there is a 1-1 correspondence between $\Omega(X)$ and $\mathcal{P}(X)$

81. Let f be a non-constant entire function and let E be the image of f. Then
1. E is an open set
2. $E \cap \{z : |z| < 1\}$ is empty
3. $E \cap \mathbb{R}$ is non-empty
4. E is a bounded set

82. Let $p(z) = z^n + a_{n-1}z^{n-1} + \cdots + a_0$, where $a_0, \ldots, a_{n-1}$ are complex numbers and let $q(z) = 1 + a_{n-1}z + \cdots + a_0 z^n$. If $|p(z)| \leq 1$ for all z with $|z| \leq 1$ then
1. $|q(z)| \leq 1$ for all z with $|z| \leq 1$
2. $q(z)$ is a constant polynomial
3. $p(z) = z^n$ for all complex numbers z
4. $p(z)$ is a constant polynomial

83. Let $f: \mathbb{C} \to \mathbb{C}$ be a holomorphic function and let u be the real part of f and v the imaginary part of f. Then, for $x, y \in \mathbb{R}$, $|f'(x+iy)|^2$ is equal to
1. $u_x^2 + u_y^2$
2. $u_x^2 + v_x^2$
3. $v_y^2 + u_y^2$
4. $v_y^2 + v_x^2$

84. Let f be an entire function. Consider $A = \{z \in \mathbb{C} \mid f^{(n)}(z) = 0$ for some positive integer $n\}$. Then
1. if $A = \mathbb{C}$, then f is a polynomial
2. if $A = \mathbb{C}$, then f is a constant function
3. if A is uncountable, then f is a polynomial
4. if A is uncountable, then f is a constant function

85. Let X and Y be topological spaces where Y is Hausdorff. Let $X \times Y$ be given the product topology. Then for a function $f: X \to Y$ which of the following statements are necessarily true?
1. if f is continuous, then $\text{graph}(f) = \{(x, f(x)) \mid x \in X\}$ is closed in $X \times Y$
2. if $\text{graph}(f)$ is closed in $X \times Y$, then f is continuous
3. if $\text{graph}(f)$ is closed in $X \times Y$, then f need not be continuous
4. if Y is finite, then f is continuous

86. Let d and d' be metrics on a non-empty set X. Then which of the following are metrics on X?
1. $\rho_1(x,y) = d(x,y) + d'(x,y)$ for all $x, y \in X$
2. $\rho_2(x,y) = d(x,y)d'(x,y)$ for all $x, y \in X$
3. $\rho_3(x,y) = \max\{d(x,y), d'(x,y)\}$ for all $x, y \in X$
4. $\rho_4(x,y) = \min\{d(x,y), d'(x,y)\}$ for all $x, y \in X$

87. Let F be a finite field and let K/F be a field extension of degree 6. Then the Galois group of K/F is isomorphic to
1. the cyclic group of order 6
2. the permutation group on $\{1, 2, 3\}$
3. the permutation group on $\{1, 2, 3, 4, 5, 6\}$
4. the permutation group on $\{1\}$

88. Let $z = e^{\frac{2\pi i}{7}}$ and let $\theta = z + z^2 + z^4$. Then
1. $\theta \in \mathbb{Q}$
2. $\theta \in \mathbb{Q}(\sqrt{D})$ for some $D > 0$
3. $\theta \in \mathbb{Q}(\sqrt{D})$ for some $D < 0$
4. $\theta \in i\mathbb{R}$

89. For any prime number p, let A_p be the set of integers $d \in \{1, 2, \ldots, 999\}$ such that the power of p in the prime factorisation of d is odd. Then the cardinality of
1. A_3 is 250
2. A_5 is 160
3. A_7 is 124
4. A_{11} is 82

90. Which of the following rings are principal ideal domains (PIDs)?
1. $\mathbb{Z}[X]/\langle X^2 + 1\rangle$
2. $\mathbb{Z}[X]$
3. $\mathbb{C}[X, Y]$
4. $\mathbb{R}[X, Y]/\langle X^2 + 1, Y\rangle$

Unit-3

91. Consider the linear system $Ax = b$ with

$$A = \begin{bmatrix} 2 & 1 & -3 \\ 1 & 2 & -2 \\ -3 & -2 & 1 \end{bmatrix}$$

Let x_n denote the n^{th} Gauss-Seidel iteration and $e_n = x_n - x$. Let M be the corresponding

matrix such that $e_{n+1} = Me_n, \quad n \geq 0$. Which of the following statements are necessarily true?
1. all eigenvalues of M have absolute value less than 1
2. there is an eigenvalue of M with absolute value at least 1
3. e_n converges to 0 as $n \to \infty$ for all $b \in \mathbb{R}^3$ and any e_0
4. e_n does not converge to 0 as $n \to \infty$ for any $b \in \mathbb{R}^3$ unless $e_0 = 0$

92. Consider the second order PDE
$$8\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} - 3\frac{\partial^2 z}{\partial y^2} = 0$$
Then which of the following are correct?
1. the equation is elliptic
2. the equation is hyperbolic
3. the general solution is $z = f\left(y - \frac{x}{2}\right) + g\left(y + \frac{3x}{4}\right)$, for arbitrary differentiable functions f and g
4. the general solution is $z = f\left(y + \frac{x}{2}\right) + g\left(y - \frac{3x}{4}\right)$, for arbitrary differentiable functions f and g

93. Consider the Lagrange equation $x^2\frac{\partial z}{\partial x} + y^2\frac{\partial z}{\partial y} = (x+y)z$. Then the general solution of the given equation is
1. $F\left(\frac{xy}{z}, \frac{x-y}{z}\right) = 0$ for an arbitrary differentiable function F
2. $F\left(\frac{x-y}{z}, \frac{1}{x} - \frac{1}{y}\right) = 0$ for an arbitrary differentiable function F
3. $z = f\left(\frac{1}{x} - \frac{1}{y}\right)$ for an arbitrary differentiable function f
4. $z = xy\, f\left(\frac{1}{x} - \frac{1}{y}\right)$ for an arbitrary differentiable function f

94. Consider a boundary value problem (BVP) $\frac{d^2y}{dx^2} = f(x)$ with boundary conditions $y(0) = y(1) = y'(1)$, where f is a real-valued continuous function on $[0,1]$. Then which of the following are true?
1. the given BVP has a unique solution for every f
2. the given BVP does not have a unique solution for some f
3. $y(x) = \int_0^x x\,t\,f(t)dt + \int_x^1 (t - x + xt) f(t)dt$ is a solution of the given BVP
4. $y(x) = \int_0^x (x - t + xt) f(t)dt + \int_x^1 xtf(t)dt$ is a solution of the given BVP

95. Consider the differential equation
$$\frac{d^2y}{dx^2} - 2\tan x\,\frac{dy}{dx} - y = 0$$
defined on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Which among the following are true?
1. there is exactly one solution $y = y(x)$ with $y(0) = y'(0) = 1$ and $y\left(\frac{\pi}{3}\right) = 2\left(1 + \frac{\pi}{3}\right)$
2. there is exactly one solution $y = y(x)$ with $y(0) = 1, y'(0) = -1$ and $y\left(-\frac{\pi}{3}\right) = 2\left(1 + \frac{\pi}{3}\right)$
3. any solution $y = y(x)$ satisfies $y''(0) = y(0)$
4. if y_1 and y_2 are any two solutions then $(ax+b)y_1 = (cx+d)y_2$ for some $a, b, c, d \in \mathbb{R}$

96. Consider a system of first order differential equations
$$\frac{d}{dt}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} x(t) + y(t) \\ -y(t) \end{bmatrix}$$
The solution space is spanned by
1. $\begin{bmatrix} 0 \\ e^{-t} \end{bmatrix}$ and $\begin{bmatrix} e^t \\ 0 \end{bmatrix}$
2. $\begin{bmatrix} e^t \\ 0 \end{bmatrix}$ and $\begin{bmatrix} \cosh t \\ e^{-t} \end{bmatrix}$
3. $\begin{bmatrix} e^{-t} \\ -2e^{-t} \end{bmatrix}$ and $\begin{bmatrix} \sinh t \\ e^{-t} \end{bmatrix}$
4. $\begin{bmatrix} e^t \\ 0 \end{bmatrix}$ and $\begin{bmatrix} e^t - \frac{1}{2}e^{-t} \\ e^{-t} \end{bmatrix}$

97. Let $B = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_1^2 + x_2^2 < 1\}$, and let $C_{Id}^2(\bar{B}; \mathbb{R}^2) = \{u \in C^2(\bar{B}; \mathbb{R}^2) \mid u(x_1, x_2) = (x_1, x_2) \text{ for } (x_1, x_2) \in \partial B\}$.
Let $u = (u_1, u_2)$ and define $J : C_{Id}^2(\bar{B}; \mathbb{R}^2) \to \mathbb{R}$ by
$$J(u) = \int_B \left(\frac{\partial u_1}{\partial x_1}\frac{\partial u_2}{\partial x_2} - \frac{\partial u_1}{\partial x_2}\frac{\partial u_2}{\partial x_1}\right) dx_1 dx_2$$
Then,
1. $\inf\{J(u) : u \in C_{Id}^2(\bar{B}; \mathbb{R}^2)\} = 0$
2. $J(u) > 0$, for all $u \in C_{Id}^2(\bar{B}; \mathbb{R}^2)$

3. $J(u) = 1$, for infinitely many $u \in C^2_{Id}(\bar{B}; \mathbb{R}^2)$
4. $J(u) = \pi$, for all $u \in C^2_{Id}(\bar{B}; \mathbb{R}^2)$

98. Let φ be the solution of the integral equation

$$\frac{1}{2}\varphi(x) - \int_0^1 e^{x-y}\varphi(y)dy = x^2 \quad 0 \le x \le 1$$

Then
1. $\varphi(0) = 20e^{-1} - 8$
2. $\varphi(0) = 20e - 8$
3. $\varphi(1) = 22 - 8e$
4. $\varphi(1) = 22 - 8e^{-1}$

99. Consider a non-zero, real-valued polynomial function $p(x) = a_0 + a_1x + a_2x^2$ of degree at most 2. Let $y = y(x)$ be a solution of the integral equation

$$y = p(x) + \int_0^x y(t)\sin(x-t)dt$$

Which of the following statements are necessarily correct?
1. $y(x)$ is a polynomial function of degree ≤ 2
2. $y(x)$ is a polynomial function of degree ≤ 4
3. If $a_1 \neq 0$ and $a_0 + 2a_2 = 0$, then $y'(0) = 0$
4. If $a_1 \neq 0$ and $a_0 + 2a_2 = 0$, then $y''(0) = 0$

100. Let $I: C^1[0,1] \to \mathbb{R}$ be defined as

$$I(u) := \frac{1}{2}\int_0^1 (u'(t)^2 - 4\pi^2 u(t)^2)dt$$

Let us set
(P) $m := \inf\{I(u): u \in C^1[0,1]: u(0) = u(1) = 0\}$
Let $\bar{u} \in C^1[0,1]$ satisfy the Euler-Lagrange Equation associated with (P). Then
1. $m = -\infty$ i.e. I is not bounded below
2. $m \in \mathbb{R}$, with $I(\bar{u}) = m$
3. $m \in \mathbb{R}$, with $I(\bar{u}) > m$
4. $m \in \mathbb{R}$, with $I(\bar{u}) < m$

101. Let $X = \{u \in C^1[0,1] \mid u(0) = 0\}$ and let $I: X \to \mathbb{R}$ be defined as

$$I(u) = \int_0^1 (u'(t)^2 - u(t)^2)dt$$

Which of the following are correct?
1. I is bounded below
2. I is not bounded below
3. I attains its infimum
4. I does not attain its infimum

102. For $f \in C[0,1]$ and $n > 1$, let $T(f) = \frac{1}{n}\left[\frac{1}{2}f(0) + \frac{1}{2}f(1) + \sum_{j=1}^{n-1} f\left(\frac{j}{n}\right)\right]$ be an approximation of the integral $I(f) = \int_0^1 f(x)dx$. For which of the following functions f is $T(f) = I(f)$?
1. $1 + \sin 2\pi nx$
2. $1 + \cos 2\pi nx$
3. $\sin^2 2\pi nx$
4. $\cos^2 2\pi(n+1)x$

UNIT-4

103. Consider a Cauchy population with probability density function

$$f_\theta(x) = \frac{1}{\pi\{1 + (x-\theta)^2\}}, -\infty < x < \infty, -\infty < \theta < \infty.$$

Let $X_1, X_2, \ldots, X_n$ be a random sample from the above population. Which of the following confidence intervals for θ have confidence coefficient $1 - \alpha$ $(0 < \alpha < 1)$?
1. $\left[X_1 - \tan\frac{\pi(1-\alpha)}{2}, X_1 + \tan\frac{\pi(1-\alpha)}{2}\right]$
2. $\left[\frac{X_1+X_2}{2} - \tan\frac{\pi(1-\alpha)}{2}, \frac{X_1+X_2}{2} + \tan\frac{\pi(1-\alpha)}{2}\right]$
3. $\left[\frac{X_1+X_2}{2} - \tan\frac{5\pi(1-\alpha)}{7}, \frac{X_1+X_2}{2} + \tan\frac{2\pi(1-\alpha)}{7}\right]$
4. $\left[\frac{X_1+X_2+X_3}{3} - \tan\frac{5\pi(1-\alpha)}{7}, \frac{X_1+X_2+X_3}{3} + \tan\frac{2\pi(1-\alpha)}{7}\right]$

104. Let $\{X_n\}$ be a sequence of independent random variables where the distribution of X_n is normal with mean μ and variance n for $n = 1,2,\ldots$ Define

$$\bar{X}_n = \frac{\sum_{i=1}^n X_i}{n} \text{ and } S_n = \sum_{i=1}^n \frac{X_i}{i} \Big/ \sum_{i=1}^n \frac{1}{i}$$

Which of the following are true?
1. $E(\bar{X}_n) = E(S_n)$ for sufficiently large n
2. $\text{Var}(S_n) < \text{Var}(\bar{X}_n)$ for sufficiently large n
3. $\bar{X}_n$ is consistent for μ
4. $\bar{X}_n$ is sufficient for μ

CENTRE FOR MATHEMATICS & STATISTICS TRIVANDRUM
Phone: 8113887329, 8075791772
Email: cmscsirnet@gmail.com

105. Let $X_1, X_2, \ldots, X_n$ be a random sample from $f_\theta(x)$, a probability density function or a probability mass function. Define $s_n^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X}_n)^2$, where $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. Then s_n^2 is unbiased for θ if

1. $f_\theta(x) = e^{-\theta}\frac{\theta^x}{x!}$, $x = 0,1,2,\ldots$ and $\theta > 0$
2. $f_\theta(x) = \frac{1}{\sqrt{2\pi}\sqrt{\theta}} e^{-\frac{x^2}{2\theta}}$, $-\infty < x < \infty$, $\theta > 0$
3. $f_\theta(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$, $x > 0$, $\theta > 0$
4. $f_\theta(x) = \theta e^{-\theta x}$, $x > 0$, $\theta > 0$

106. For $n \geq 1$, let X_n be a Poisson random variable with mean n^2. Which of the following are equal to

$$\frac{1}{\sqrt{2\pi}}\int_2^\infty e^{-x^2/2}\,dx$$

1. $\lim_{n\to\infty} P\{X_n > (n+1)^2\}$
2. $\lim_{n\to\infty} P\{X_n \leq (n+1)^2\}$
3. $\lim_{n\to\infty} P\{X_n < (n-1)^2\}$
4. $\lim_{n\to\infty} P\{X_n < (n-2)^2\}$

107. Which of the following are correct?

1. if X and Y are $N(0,1)$ then $\frac{X+Y}{\sqrt{2}}$ is $N(0,1)$
2. if X and Y are independent $N(0,1)$ then $\frac{X}{Y}$ has t-distribution
3. if X and Y are independent Uniform$(0,1)$ then $\frac{X+Y}{2}$ is Uniform$(0,1)$
4. if X is Binomial(n,p) then $n - X$ is Binomial$(n, 1-p)$

108. Consider a Markov chain with five states $\{1,2,3,4,5\}$ and transition matrix

$$P = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{7} & 0 & 0 & \frac{6}{7} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 \\ 0 & \frac{5}{8} & 0 & 0 & \frac{3}{8} \end{pmatrix}$$

Which of the following are true?

1. 3 and 1 are in the same communicating class
2. 1 and 4 are in the same communicating class
3. 4 and 2 are in the same communicating class
4. 2 and 5 are in the same communicating class

109. Let S be the set of all 3×3 matrices having 3 entries equal to 1 and 6 entries equal to 0. A matrix M is picked uniformly at random from the set S. Then

1. $P\{M \text{ is nonsingular}\} = \frac{1}{14}$
2. $P\{M \text{ has rank } 1\} = \frac{1}{14}$
3. $P\{M \text{ is identity}\} = \frac{1}{14}$
4. $P\{\text{trace}(M) = 0\} = \frac{1}{14}$

110. Suppose A, B, C are events in a common probability space with
$P(A) = 0.2$ $P(B) = 0.2$ $P(C) = 0.3$
$P(A \cap B) = 0.1$ $P(A \cap C) = 0.1$
$P(B \cap C) = 0.1$
Which of the following are possible values of $P(A \cup B \cup C)$?

1. 0.5
2. 0.3
3. 0.4
4. 0.9

111. Consider an $M/M/1$ queue with interarrival time having exponential distribution with mean $\frac{1}{\lambda}$ and service time having exponential distribution with mean $\frac{1}{\mu}$. Which of the following are true?

1. if $0 < \lambda < \mu$ then the queue length has limiting distribution Poisson $(\mu - \lambda)$
2. if $0 < \mu < \lambda$ then the queue length has limiting distribution Poisson $(\lambda - \mu)$
3. if $0 < \lambda < \mu$ then the queue length has limiting distribution which is geometric
4. if $0 < \mu < \lambda$ then the queue length has limiting distribution which is geometric

112. Arrival of customers in a shop is a Poisson process with intensity $\lambda = 2$. Let X be the number of customers entering during the time interval $(1, 2)$ and let Y be the number of customers entering during the time interval $(5, 10)$. Which of the following are true?

1. $P(X = 0 \mid X + Y = 12) = \left(\frac{5}{6}\right)^{12}$
2. X and Y are independent
3. $X + Y$ is Poisson with parameter 6
4. $X - Y$ is Poisson with parameter 8

CENTRE FOR MATHEMATICS & STATISTICS TRIVANDRUM
Phone: 8113887329, 8075791772
Email: cmscsirnet@gmail.com

113. Twenty identical items are put in a life testing experiment starting at time 0. The failure times of the items are recorded in a sequential manner. The experiment stops if all the items fail or a pre-fixed time $T > 0$ is reached, whichever is earlier. If the lifetimes of the items are independent identically distributed exponential random variables with mean θ, where $0 < \theta \leq 10$, then which of the following statements are correct?

1. the MLE of θ always exists
2. the MLE of θ may not exist
3. the MLE of θ is an unbiased estimator of θ, if it exists
4. the MLE of θ is bounded with probability 1, if it exists

114. Consider a BIBD(v, b, r, k, λ) with $k = 5$. Let $N = ((n_{ij}))$ be the incidence matrix, where n_{ij} = number of times ith treatment appears in jth block, $1 \leq i \leq v, 1 \leq j \leq b$. Let $C = rI - \frac{1}{k}NN^t$. Which of the following are true?

1. C has a characteristic root 0
2. rank of N is v
3. the above BIBD is connected
4. trace of the C is $4b$

115. Suppose there are k groups each consisting of N boys. We want to estimate the mean age μ of these kN boys. Fix $1 < n < N$ and consider the following two sampling schemes.

I. Draw a simple random sample without replacement of size kn out of all kN boys.
II. From each of the k groups draw a simple random sample with replacement of size n.

Let $\overline{Y}$ and $\overline{Y_G}$ be the respective sample mean ages for the two schemes. Which of the following are true?

1. $E(\overline{Y}) = \mu$
2. $E(\overline{Y_G}) = \mu$
3. $\text{Var}(\overline{Y})$ may be less than $\text{Var}(\overline{Y_G})$ in some cases
4. $\text{Var}(\overline{Y}) = \text{Var}(\overline{Y_G})$ if all the group means are same

116. Suppose $X_1, \ldots, X_n$ are i.i.d. random vectors from $N_p(0, \Sigma)$. Let $\ell \in \mathbb{R}^p$, $E\left(\sum_{i=1}^n \ell^t X_i X_i^t \ell\right) = c$ and $E\left(\sum_{i=1}^n X_i X_i^t\right) = A$

Which of the following statements are necessarily true?

1. $c = \ell^t \ell$
2. $\ell^t\left(\sum_{i=1}^n X_i X_i^t\right)\ell$ follows a chi-squared distribution
3. $\ell^t\left(\sum_{i=1}^{n_1} X_i X_i^t\right)\ell$ and $\ell^t\left(\sum_{i=n_1+1}^{n} X_i X_i^t\right)\ell$ are independently distributed for $1 \leq n_1 \leq n-1$.
4. $A = \Sigma$

117. Suppose we have a random sample of size n $(n \geq 1)$ from the density

$$f_\lambda(x) = \begin{cases} 2\lambda x e^{-\lambda x^2}, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

If the prior of λ is an exponential distribution with mean 1, then which of the following statements are correct?

1. the posterior distribution of λ is an exponential distribution
2. the Bayes estimator of λ w.r.t. the squared error loss function exists and is unique
3. the Bayes estimator of λ w.r.t. the absolute error loss function exists and is unique
4. the Bayes estimator of $e^{-\lambda}$ does not exist

118. Let $X_1, \ldots, X_n$ be a random sample from $N(\theta, 1)$, $\theta \in [-100, 100]$ and let $Y_1, \ldots, Y_n$ be defined by

$$Y_i = \begin{cases} 0 & \text{if } X_i < 0 \\ 1 & \text{if } X_i \geq 0 \end{cases}$$

Suppose $\hat{\theta}_n$ and $\tilde{\theta}_n$ denote the MLEs of θ based on $\{X_1, \ldots, X_n\}$ and on $\{Y_1, \ldots, Y_n\}$, respectively. Which of the following statements are true?

1. $\lim_{n\to\infty} E(\hat{\theta}_n) = \theta$
2. $\lim_{n\to\infty} E(\tilde{\theta}_n) = \theta$
3. $\hat{\theta}_n$ is a consistent estimator of θ
4. $\tilde{\theta}_n$ is a consistent estimator of θ

119. Consider the following regression problem
$y_i = \beta_1 e^i + \beta_2 e^{-i} + \epsilon_i;\ i = 1, \ldots, n.$
Here $\epsilon_1, \ldots, \epsilon_n$ are i.i.d. $N(0, \sigma^2)$ random variables. If $\hat{\beta}_1$ and $\hat{\beta}_2$ are the least square estimators of β_1 and β_2, respectively, then which of the following statements are correct?

1. $E(\hat{\beta}_1) = \beta_1$
2. $E(\hat{\beta}_2) = \beta_2$
3. $\text{Var}(\hat{\beta}_1) > \text{Var}(\hat{\beta}_2)$
4. $\text{Cov}(\hat{\beta}_1, \hat{\beta}_2) < 0$

120. Let $X_1, X_2, \ldots, X_n$ be a random sample from an unknown continuous distribution function F with median θ. Let T_n count the number of i for which $X_i > 0$. Consider the problem of testing $H_0: \theta = 0$ against $H_1: \theta = -1$ based on the test statistic T_n. Which of the following are true?

1. the distribution of T_n is independent of F under H_1
2. left-tailed test based on T_n is consistent against H_1
3. left-tailed test based on T_n is unbiased against H_1
4. left-tailed test based on T_n has the p-value $P[T_n \leq \text{observed } T_n]$ under H_1

FOR ROUGH WORK

CENTRE FOR MATHEMATICS & STATISTICS TRIVANDRUM
Phone: 8113887329, 8075791772
Email: cmscsirnet@gmail.com

BOOKLET CODE B	
Q. No.	**Key**
1	3
2	1
3	2
4	4
5	1
6	2
7	2
8	4
9	3
10	2
11	4
12	4
13	4
14	4
15	3
16	3
17	3
18	1
19	3
20	3
21	2
22	3
23	2
24	2
25	3
26	2
27	3
28	4
29	4
30	4
31	3
32	2
33	3
34	4
35	3
36	4
37	2
38	1
39	2
40	2
41	1
42	3
43	3
44	3
45	3
46	1
47	1
48	4
49	2
50	1
51	4
52	2
53	3
54	1
55	3
56	4
57	3
58	2
59	4
60	2
61	4
62	1,2,4
63	3,4
64	3,4
65	1,3,4
66	1
67	1,4
68	3,4
69	1,2,3
70	1,2,3
71	2
72	2,3,4
73	1,2,3
74	1,2,3
75	1,4
76	2,3
77	3,4
78	1,4
79	3
80	1,2,3,4
81	1,3
82	1,2,3
83	1,2,3,4
84	1,3
85	1,3
86	1,3
87	1
88	3,4
89	1,3,4
90	1,4
91	2,4
92	2,3
93	1,2,4
94	1,3
95	1,2,3,4
96	3,4
97	2,4
98	1,3
99	2,4
100	1
101	1,3
102	1,4
103	1,2,3,4
104	1,2
105	1,2
106	1,3
107	2,4
108	2,4
109	1,2
110	1,3
111	3
112	1,2
113	1,4
114	1,2,3,4
115	1,2,3
116	3
117	2,3
118	1,2,3,4
119	1,2,4
120	2,3