Part A Mathematical Sciences

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Mandatory or Optional:	Mandatory
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Group All Questions:	No

The number of triangles in the figure is

(1) 9

(2) 10

(3) 11

(4) 12

There are nine identical balls, one of which is heavier than the other eight. What is the least number of weighings, using a two-pan balance, needed for definitely identifying the heavier ball?

(1) One

(2) Two

(3) Three

(4) Four

The difference, the sum and the product of two integers are in the proportion 1:3:10. The two integers are:

(1) 3, 9

(2) 2, 5

(3) 5, 10

(4) 3, 10

The trends of two quantities over five years are shown is the graph. Which of the following are valid inferences?

A. The mean values of the quantities are nearly equal

B. The variations in the two quantities are nearly equal

C. Quantity 1 varies less over the given period as compared to Quantity 2

(1) Only A is true

(2) Only B is true

(3) A and C are true

(4) A and B are true

In a population of 900, the number of married couples is as much as the number of singles. There are 100 twins of which 50 twins are singles. The population has 400 females in all. What is the number of married persons?

(1) 325

(2) 600

(3) 250

(4) 300

In a certain cipher language 'BIKE' is coded as 'YFHB' and 'CAR' is coded as 'ZXO' then 'SCOOTER' can be coded as

(1) TAPPIYB

(2) PYVVAHJ

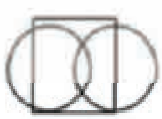
(3) PZLLQBO

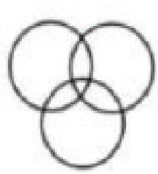
(4) JZKKMCO

Which among the following diagrams can represent the relationships between houses, offices and buildings?

(1) 

(2) 

(3) 

(4) 

Consider a location on the Earth where the Sun is overhead at noon. Compared to its shadow at 10.00 AM, the shadow of a tower at 4.00 PM would be

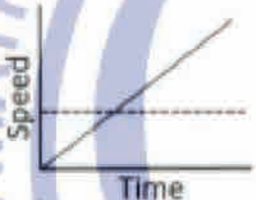
(1) twice longer

(2) three times longer

(3) four times longer

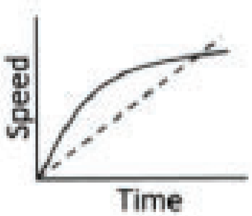
(4) eight times longer

A girl is running at constant speed to catch a bus which is stationary. Before she reaches the bus, the bus leaves and moves with a constant acceleration. Which one of these graphs describes the situation correctly?

(1) 

(2) 

(3) 

(4) 

A dart is randomly thrown at a circular board on which two concentric rings of radii R and 2R having the same width (width much less than R) are marked. The probability of the dart hitting the smaller ring is

(1) twice the probability that it hits the larger ring.

(2) half of the probability that it hits the larger ring.

(3) four times the probability that it hits the larger ring.

(4) one-fourth the probability that it hits the larger ring.

The length of a rod is measured repeatedly by two persons. Person A reports the length to be 1002 ± 1 cm while person B reports the length to be 1001 ± 2 cm. It is known from a more reliable method that the length is 1000.1 ± 0.5 cm. Which one of the following statements is correct?

(1) Measurements made by B are less accurate, but more precise, compared to those by A.

(2) Measurements made by A are less accurate, but more precise, compared to those by B.

(3) Measurements made by B are more precise and more accurate, compared to those by A.

(4) Measurements made by A are more precise and more accurate, compared to those by B.

Seven chairs numbered 1 to 7 are placed around a round table. Starting from chair number 5, a person keeps going around the table anticlockwise. After crossing 41 chairs, the person will reach the chair number

(1) 1

(2) 3

(3) 5

(4) 7

A, B, C and D are four consecutive points on a circle such that chords AB=BC=CD=10.0 cm and DA = 20.0 cm. The radius of the circle (in cm) is

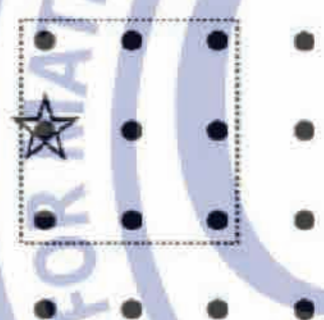
(1) 10.0

(2) $10\sqrt{2}$

(3) $10\sqrt{3}$

(4) 20.0

A move of a coin is defined as crossing any number of points in a straight line on the 4×4 grid (horizontally, vertically or diagonally). What is the least number of moves in which a coin, starting from the indicated position, can cover all nine points within the marked square?



(1) four

(2) five

(3) six

(4) seven

A tells B, "I could be visiting you on any day in the next two months and you must give me gold coins of as much total weight in grams as the number of days that would elapse from today". If gold coins are available in integer gram weights, what is the least number of coins with which B can meet A's demand on any day?

(1) 31

(2) 7

(3) 6

(4) 13

The graph below shows the rainfall and temperature at a place over one week. Which day of the week would feel the most humid?

(1) Monday

(2) Wednesday

(3) Thursday

(4) Saturday

The difference between the squares of two consecutive integers is 408235. The sum of the numbers is

(1) 16324

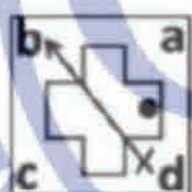
(2) 27061

(3) 180235

(4) 408235

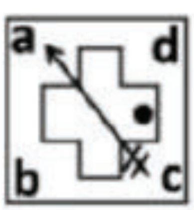
Find out the next figure in following sequence?

(1)



(2)

(3)



A partially filled hour glass has water falling from the upper bowl to the lower bowl. Which of the following statements is correct?

(1) The level of water rises in the lower bowl at the same rate as the fall in the upper bowl

(2) The level of water rises in the lower bowl at the half rate as the fall in the upper bowl

(3) The rate of increase in the volume of water in the lower bowl is the same as the rate of decrease in the upper bowl

(4) The area of top surface of the water column is the same in both bowls at all times

Part B Mathematical Sciences

Section Id :	18798044
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Which of the following sets is countable?

(1) The set of all functions from $\mathbb{Q}$ to $\mathbb{Q}$

(2) The set of all functions from $\mathbb{Q}$ to $\{0, 1\}$

(3) The set of all functions from $\mathbb{Q}$ to $\{0, 1\}$ which vanish outside a finite set

(4) The set of all subsets of $\mathbb{N}$

Examine the following statements:

a. Fat cells normally produce hormone A in proportion to the amount of fat. Obese individuals, however, have lower than normal levels of hormone A.

b. Hormone A reduces food intake.

Which among the following is a valid inference based on the above statements?

(1) Impaired production of hormone A causes obesity

(2) Impaired action of hormone A causes obesity

(3) Obesity results into low levels of hormone A

(4) Excess food intake causes depletion of hormone A

Let $(x_n)_{n\geq 1}$ be a sequence of non-negative real numbers. Then, which of the following is true?

(1) $\liminf x_n = 0 \Rightarrow \lim x_n^2 = 0$

(2) $\limsup x_n = 0 \Rightarrow \lim x_n^2 = 0$

(3) $\liminf x_n = 0 \Rightarrow (x_n)_{n\geq 1}$ is bounded

(4) $\liminf x_n^2 > 4 \Rightarrow \limsup x_n > 4$

Let $\leq$ be the usual order on the field $\mathbb{R}$ of real numbers. Define an order $\preccurlyeq$ on $\mathbb{R}^2$ by $(a,b) \preccurlyeq (c,d)$ if $(a < c)$, or $(a = c$ and $b \leq d)$. Consider the subset $E = \left\{\left(\frac{1}{n}, 1-\frac{1}{n}\right) \in \mathbb{R}^2 : n \in \mathbb{N}\right\}$. With respect to $\preccurlyeq$ which of the following statements is true?

(1) $\inf(E) = (0,1)$ and $\sup(E) = (1,0)$

(2) $\inf(E)$ does not exist but $\sup(E) = (1,0)$

(3) $\inf(E) = (0,1)$ but $\sup(E)$ does not exist

(4) Both $\inf(E)$ and $\sup(E)$ do not exist

What is the sum of the following series?

$$\left(\frac{1}{2\cdot 3}+\frac{1}{2^2\cdot 3}\right)+\left(\frac{1}{2^2\cdot 3^2}+\frac{1}{2^3\cdot 3^2}\right)+\cdots+\left(\frac{1}{2^n\cdot 3^n}+\frac{1}{2^{n+1}\cdot 3^n}\right)+\cdots$$

(1) $\frac{3}{8}$

(2) $\frac{3}{10}$

(3) $\frac{3}{14}$

(4) $\frac{3}{16}$

Let $X \subset \mathbb{R}$ be an infinite countable bounded subset of $\mathbb{R}$. Which of the following statements is true?

(1) X cannot be compact

(2) X contains an interior point

(3) X may be closed

(4) closure of X is countable

Let $E = \left\{\frac{1}{n} \mid n \in \mathbb{N}\right\}$. For each $m \in \mathbb{N}$ define $f_m: E \to \mathbb{R}$ by

$$f_m(x) = \begin{cases} \cos(mx) \text{ if } x \geq \frac{1}{m} \\ 0 \text{ if } \frac{1}{m+10} < x < \frac{1}{m} \\ x \text{ if } x \leq \frac{1}{m+10} \end{cases}$$

Then which of the following statements is true?

(1) No subsequence of $(f_m)_{m\geq 1}$ converges at every point of E

(2) Every subsequence of $(f_m)_{m\geq 1}$ converges at every point of E

(3) There exist infinitely many subsequences of $(f_m)_{m\geq 1}$ which converge at every point of E

(4) There exists a subsequence of $(f_m)_{m\geq 1}$ which converges to 0 at every point of E

Let $M_4(\mathbb{R})$ be the space of all (4×4) matrices over $\mathbb{R}$. Let

$$W = \left\{(a_{ij}) \in M_4(\mathbb{R}) \mid \sum_{i+j=k} a_{ij} = 0, \text{for } k = 2,3,4,5,6,7,8\right\}$$

Then $\dim(W)$ is

(1) 7

(2) 8

(3) 9

(4) 10

Let $A = \begin{pmatrix} 2 & 0 & 5 \\ 1 & 2 & 3 \\ -1 & 5 & 1 \end{pmatrix}$. The system of linear equations $AX = Y$ has a solution

(1) only for $Y = \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}$, $x \in \mathbb{R}$

(2) only for $Y = \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix}$, $y \in \mathbb{R}$

(3) only for $Y = \begin{pmatrix} 0 \\ y \\ z \end{pmatrix}$, $y, z \in \mathbb{R}$

(4) for all $Y \in \mathbb{R}^3$

For $t \in \mathbb{R}$, define

$M(t) = \begin{pmatrix} 1 & t & 0 \\ 1 & 1 & t^2 \\ 0 & 1 & 1 \end{pmatrix}$.

Then which of the following statements is true?

(1) $\det M(t)$ is a polynomial function of degree 3 in t

(2) $\det M(t) = 0$ for all $t \in \mathbb{R}$

(3) $\det M(t)$ is zero for infinitely many $t \in \mathbb{R}$

(4) $\det M(t)$ is zero for exactly two $t \in \mathbb{R}$

For a quadratic form in 3 variables over $\mathbb{R}$, let r be the rank and s be the signature. The number of possible pairs (r, s) is

(1) 13

(2) 9

(3) 10

(4) 16

Let V be a vector space of dimension 3 over $\mathbb{R}$. Let $T: V \to V$ be a linear transformation, given by the matrix $A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & -4 & 3 \\ -2 & 5 & -3 \end{pmatrix}$ with respect to an ordered basis $\{v_1, v_2, v_3\}$ of V. Then which of the following statements is true?

(1) $T(v_3) = 0$

(2) $T(v_1 + v_2) = 0$

(3) $T(v_1 + v_2 + v_3) = 0$

(4) $T(v_1 + v_3) = T(v_2)$

Let $C[0, 1]$ be the space of continuous real valued functions on $[0, 1]$. Define $\langle f, g \rangle = \int_0^1 f(t)(g(t))^2 dt$ for all $f, g \in C[0, 1]$

Then which of the following statements is true?

(1) $\langle \, , \, \rangle$ is an inner product on $C[0, 1]$

(2) $\langle \, , \, \rangle$ is a bilinear form on $C[0, 1]$ but is not an inner product on $C[0, 1]$

(3) $\langle \, , \, \rangle$ is not a bilinear form on $C[0, 1]$

(4) $\langle f, f \rangle \geq 0$ for all $f \in C[0, 1]$

Let $T: \mathbb{C} \to \mathbb{M}_2(\mathbb{R})$ be the map given by

$$T(z) = T(x + iy) = \begin{bmatrix} x & y \\ -y & x \end{bmatrix}$$

Then which of the following statements is false?

(1) $T(z_1 z_2) = T(z_1)T(z_2)$ for all $z_1,\ z_2 \in \mathbb{C}$

(2) $T(z)$ is singular if and only if $z = 0$

(3) There does not exist non-zero $A \in \mathbb{M}_2(\mathbb{R})$ such that the trace of $T(z)A$ is zero for all $z \in \mathbb{C}$

(4) $T(z_1 + z_2) = T(z_1) + T(z_2)$ for all $z_1,\ z_2 \in \mathbb{C}$

For $z \in \mathbb{C}$, let $f(z) = \begin{cases} \frac{\bar{z}^2}{z} & \text{if } z \neq 0, \\ 0 & \text{otherwise.} \end{cases}$

Then which of the following statements is false?

(1) $f(z)$ is continuous everywhere

(2) $f(z)$ is not analytic in any open neighbourhood of zero

(3) $zf(z)$ satisfies the Cauchy-Riemann equations at zero

(4) $f(z)$ is analytic in some open subset of $\mathbb{C}$

Consider the polynomial $f(z) = z^2 + az + p^{11}$, where $a \in \mathbb{Z}\backslash\{0\}$ and $p \geq 13$ is a prime. Suppose that $a^2 \leq 4p^{11}$. Which of the following statements is true?

(1) f has a zero on the imaginary axis

(2) f has a zero for which the real and imaginary parts are equal

(3) f has distinct roots

(4) f has exactly one real root

Let $f: \mathbb{C} \to \mathbb{C}$ be an entire function with $f\left(\frac{1}{n}\right) = \frac{1}{n^2}$ for all $n \in \mathbb{N}$. Then which of the following statements is true?

(1) No such f exists

(2) such an f is not unique

(3) $f(z) = z^2$ for all $z \in \mathbb{C}$

(4) f need not be a polynomial function

Let G be a group of order p^n, p a prime number and $n > 1$. Then which of the following is true?

(1) Centre of G has at least two elements

(2) G is always an Abelian group

(3) G has exactly two normal subgroups (i.e., G is a simple group)

(4) If H is any other group of order p^n, then G is isomorphic to H

A permutation σ of $[n] = \{1, 2, \cdots, n\}$ is called irreducible, if the restriction $\sigma|_{[k]}$ is not a permutation of $[k]$ for any $1 \leq k < n$. Let a_n be the number of irreducible permutations of $[n]$. Then $a_1 = 1, a_2 = 1$ and $a_3 = 3$. The value of a_4 is

(1) 12

(2) 13

(3) 14

(4) 15

Let S_5 be the symmetric group on five symbols. Then which of the following statements is false?

(1) S_5 contains a cyclic subgroup of order 6

(2) S_5 contains a non-Abelian subgroup of order 8

(3) S_5 does not contain a subgroup isomorphic to $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$

(4) S_5 does not contain a subgroup of order 7

Let X be an infinite set. Consider the topology τ on X whose non-empty open sets are complements of finite sets. Then which of the following statements is true?

(1) X is disconnected

(2) X is compact

(3) No sequence in X converges in X

(4) Every sequence in X converges to a unique point in X

For the following system of ordinary differential equations

$\frac{dx}{dt} = x(3 - 2x - 2y),$

$\frac{dy}{dt} = y(2 - 2x - y),$

the critical point $(0, 2)$ is

(1) a stable spiral

(2) an unstable spiral

(3) a stable node

(4) an unstable node

Consider the system of ordinary differential equations

$\frac{dx}{dt} = 4x^3y^2 - x^5y^4,$

$\frac{dy}{dt} = x^4y^5 + 2x^2y^3.$

Then for this system there exists

(1) a closed path in $\{(x, y) \in \mathbb{R}^2 | x^2 + y^2 \leq 5\}$

(2) a closed path in $\{(x, y) \in \mathbb{R}^2 | 5 < x^2 + y^2 \leq 10\}$

(3) a closed path in $\{(x, y) \in \mathbb{R}^2 | x^2 + y^2 > 10\}$

(4) no closed path in $\mathbb{R}^2$

Let $u(x, y)$ be the solution of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 64$ in the unit disc $\{(x, y) | x^2 + y^2 < 1\}$ and such that u vanishes on the boundary of the disc. Then $u\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$ is equal to

(1) 7

(2) 16

(3) -7

(4) -16

The Cauchy problem

$y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y} = 0$

and $x_0(s) = \cos(s), y_0(s) = \sin(s), z_0(s) = 1,\ s > 0$ has

(1) a unique solution

(2) no solution

(3) more than one but finite number of solutions

(4) infinitely many solutions

Let $x = \xi$ be a solution of $x^4 - 3x^2 + x - 10 = 0$. The rate of convergence for the iterative method $x_{n+1} = 10 - x_n^4 + 3x_n^2$ is equal to

(1) 1

(2) 2

(3) 3

(4) 4

Let $y = \phi(x)$ be the extremizing function for the functional $I(y) = \int_0^1 y^2 \left(\frac{dy}{dx}\right)^2 dx$,

subject to $y(0) = 0,\ y(1) = 1$. Then $\phi(1/4)$ is equal to

(1) 1/2

(2) 1/4

(3) 1/8

(4) 1/12

Let ϕ be the solution of

$\phi(x) = 1 - 2x - 4x^2 + \int_0^x [3 + 6(x - t) - 4(x - t)^2]\phi(t)dt.$

Then $\phi(1)$ is equal to

(1) e^{-1}

(2) e^{-2}

(3) e

(4) e^2

Consider a mass-less infinite straight wire with one end fixed at O. Assume that the wire is rotating in a plane about the point O with constant angular velocity ω. Consider a bead of mass m sliding along the wire in the absence of external forces. Let $r(t)$ denote the distance of the bead from O at time $t \geq 0$, and $\frac{dr}{dt}(0) = 0$. Then which of the following statements is true?

(1) $\exists M > 0, \alpha > 0$ such that $r(t) > Me^{\alpha t},\ t > 0$

(2) $r(t) \to 0$ as $t \to \infty$

(3) $r(t)$ is a constant function

(4) $\frac{dr(t)}{dt}$ changes its sign for some $t > 0$

Let X_1 and X_2 be a random sample of size 2 from Uniform $[0, \theta]$ distribution, $\theta > 0$. Define $M = \max\{X_1, X_2\}$. What is the confidence coefficient of the confidence interval $\left[\frac{3}{7}M, 2M\right]$ for θ?

(1) 0.6285

(2) 0.753

(3) 0.8333

(4) 0.95

Let X be a real-valued random variable such that $E[e^X] < \infty$ and $E[e^X] = e^{E[X]}$. Then which of the following is correct?

(1) $P(X \geq a) \geq e^{E[X]-a}$ for all $a \in \mathbb{R}$

(2) $E[X^3] = (E[X])^3$

(3) $\mathrm{Var}(X) = 0$

(4) $X \geq 0$ almost surely

There are three urns U_1, U_2, U_3, each with balls of two colours. U_1 contains 2 white balls and 3 black balls, U_2 contains 3 white balls and 2 black balls and U_3 contains 5 white balls and 5 black balls. An urn is chosen at random and a ball is drawn from that urn at random. What is the probability that U_2 was chosen given that the ball picked is black in colour?

(1) $\frac{1}{3}$

(2) $\frac{4}{15}$

(3) $\frac{2}{15}$

(4) $\frac{1}{6}$

Let $\{X_n : n \geq 0\}$ be a two state Markov chain with state space $S = \{0, 1\}$ and transition matrix

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

Assuming $X_0 = 0$, the expected return time to 0 is

(1) $\frac{5}{2}$

(2) $\frac{9}{4}$

(3) $\frac{3}{2}$

(4) 3

Let X and Y be independent Exponential random variables with means $\frac{1}{\lambda}$ and $\frac{1}{\mu}$ respectively with $\lambda \neq \mu$. Let $f_Z(z)$ denote the density function of $Z = X + Y$. Then for $z > 0$,

(1) $f_Z(z) = (\lambda + \mu)e^{-(\lambda+\mu)z}$

(2) $f_Z(z) = \frac{\lambda\mu}{\lambda+\mu} e^{\frac{-\lambda\mu}{\lambda+\mu}z}$

(3) $f_Z(z) = \frac{\lambda\mu}{\lambda-\mu}\left(e^{-\mu z} - e^{-\lambda z}\right)$

(4) $f_Z(z) = \begin{cases} \frac{\lambda\mu}{\lambda-\mu} e^{\frac{-\lambda\mu}{\lambda-\mu}z} \; if \; \lambda > \mu \\ \frac{\lambda\mu}{\mu-\lambda} e^{\frac{-\lambda\mu}{\mu-\lambda}z} \; if \; \mu > \lambda \end{cases}$

Let $X_1, X_2, \cdots, X_n$ $(n \geq 2)$ be a random sample from a distribution with probability density function f_θ, $\theta > 0$, unknown, where

$$f_\theta(x) = \begin{cases} \frac{2(\theta - x)}{\theta^2}, 0 \leq x \leq \theta, \\ 0, \quad \text{otherwise.} \end{cases}$$

Let $\bar{X}_n$ be the sample mean and $X_{(n)} = max\{X_1, X_2, \cdots, X_n\}$.

Then which of the following statements is correct?

(1) $X_{(n)}$ is sufficient for θ

(2) $X_{(n)}$ is unbiased for θ

(3) $3\bar{X}_n$ is unbiased for θ

(4) $3\bar{X}_n$ is sufficient for θ

Suppose $\binom{X_1}{Y_1}, \binom{X_2}{Y_2}, \binom{X_3}{Y_3}$ are i.i.d. observations from the Uniform distribution on the unit square$[0, 1] \times [0, 1]$. What is the probability that the rank correlation between the X_i and the Y_i values is 1?

(1) 0

(2) $\frac{1}{2}$

(3) $\frac{1}{3}$

(4) $\frac{1}{6}$

Suppose data $(X_1, Y_1), (X_2, Y_2), \cdots, (X_n, Y_n)$ are generated as follows : $Y_1, Y_2, \cdots, Y_n \sim$ i. i. d. Bernoulli $\left(\frac{1}{2}\right)$ and $X_i | Y_i = y \sim$ Uniform $(0, y+1)$. Define

$$h(x) = \begin{cases} 1 & \text{if } x \geq 1 \\ 0 & \text{otherwise.} \end{cases}$$

Then, which of the following is a correct linear regression model for $m(x) = E(Y_i | X_i = x)$, in the sense that the true $m(x)$ is obtained for some values of the parameters $\alpha_0, \alpha_1, \alpha_2$ for all x?

(1) $m(x) = \alpha_0 + \alpha_1 x$

(2) $m(x) = \alpha_0 + \alpha_1 x + \alpha_2 x^2$

(3) $m(x) = \alpha_0 + \alpha_1 x + \alpha_2 x\, h(x)$

(4) $m(x) = \alpha_0 + \alpha_1 x + \alpha_2 h(x)$

Let $(X_1, Y_1), (X_2, Y_2) \cdots (X_n, Y_n), n \geq 5$, be a random sample from a Bivariate Normal $(\mu_1, \mu_2, \sigma_1, \sigma_2, \rho)$ distribution with all parameters unknown. For testing $H_0: \rho = 0$ against $H_1: \rho \neq 0$ if you use the usual t-test and your observed sample correlation coefficient is 0, then what is the p-value?

(1) 0

(2) 0.05

(3) 0.5

(4) 1

To draw a sample of size $n (\geq 5)$ using a without replacement scheme from a finite population $\{U_1, U_2, \ldots, U_N\}$ of size N, the first unit is chosen using $PPS(p_1, p_2, \ldots, p_N)$ scheme and the remaining $(n-1)$ units are drawn using SRSWOR. Then the probability that U_2 is included in the sample is

(1) $\frac{N-n}{N-1} p_2 + \frac{1}{N-1}$

(2) $\frac{N-n}{N-1} p_2 + \frac{n-2}{N-1}$

(3) $\frac{N-n}{N-1} p_2 + \frac{N-n}{N-1}$

(4) $\frac{N-n}{N-1} p_2 + \frac{n-1}{N-1}$

In a 2^3 factorial design, the treatment combinations of three treatments A, B and C are allotted to 2 blocks of 4 plots each. Suppose the key block is as follows.

Key block: (1), a, bc, abc

Then the confounded treatment combination is

(1) AB

(2) AC

(3) BC

(4) ABC

Subject to the conditions

$0 \leq x \leq 10,\ 0 \leq y \leq 5,$

the minimum value of the function $4x - 5y + 10$ is

(1) 10

(2) 0

(3) -25

(4) -15

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Section Number :	3
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Group All Questions:	No

Let n be a fixed natural number. Then the series

$$\sum_{m \geq n} \frac{(-1)^m}{m} \text{ is}$$

(1) Absolutely convergent

(2) Divergent

(3) Absolutely convergent if $n > 100$

(4) Convergent

For each natural number $n \geq 1$, let $a_n = \frac{n}{10^{\lceil \log_{10} n \rceil}}$,

where $\lceil x \rceil$ =smallest integer greater than or equal to x. Which of the following statements are true?

(1) $\liminf_{n\to\infty} a_n = 0$

(2) $\liminf_{n\to\infty} a_n$ does not exist

(3) $\liminf_{n\to\infty} a_n = 0.15$

(4) $\limsup_{n\to\infty} a_n = 1$

Let $\{a_n\}_{n\geq 1}$ be a bounded sequence of real numbers. Then

(1) Every subsequence of $\{a_n\}_{n\geq 1}$ is convergent

(2) There is exactly one subsequence of $\{a_n\}_{n\geq 1}$ which is convergent

(3) There are infinitely many subsequences of $\{a_n\}_{n\geq 1}$ which are convergent

(4) There is a subsequence of $\{a_n\}_{n\geq 1}$ which is convergent

Let $f: [0,1] \to \mathbb{R}$ be a monotonic function with $f\left(\frac{1}{4}\right) f\left(\frac{3}{4}\right) < 0$.

Suppose $sup\{x \in [0,1]: f(x) < 0\} = \alpha$.

Which of the following statements are correct?

(1) $f(\alpha) < 0$

(2) If f is increasing, then $f(\alpha) \leq 0$

(3) If f is continuous and $\frac{1}{4} < \alpha < \frac{3}{4}$, then $f(\alpha) = 0$

(4) If f is decreasing, then $f(\alpha) < 0$

Let $f(x)$ be a real polynomial of degree 4. Suppose $f(-1) = 0, f(0) = 0,$

$f(1) = 1$ and $f^{(1)}(0) = 0$, where $f^{(k)}(a)$ is the value of k^{th} derivative of $f(x)$ at $x = a$. Which of the following statements are true?

(1) There exists $a \in (-1,1)$ such that $f^{(3)}(a) \geq 3$

(2) $f^{(3)}(a) \geq 3$ for all $a \in (-1,1)$

(3) $0 < f^{(3)}(0) \leq 2$

(4) $f^{(3)}(0) \geq 3$

Let $N \geq 5$ be an integer. Then which of the following statements are true?

(1) $\sum_{n=1}^{N} \frac{1}{n} \leq 1 + \log N$

(2) $\sum_{n=1}^{N} \frac{1}{n} < 1 + \log N$

(3) $\sum_{n=1}^{N} \frac{1}{n} \leq \log N$

(4) $\sum_{n=1}^{N} \frac{1}{n} \geq \log N$

Let $L^2([-\pi,\pi])$ be the metric space of Lebesgue square integrable functions on $[-\pi,\pi]$ with a metric d given by

$$d(f,g) = \left[\int_{-\pi}^{\pi} (f(x) - g(x))^2 dx\right]^{1/2} \text{ for } f, g \in L^2([-\pi,\pi])$$

Consider the subset

$S = \{\sin(2^n x) : n \in \mathbb{N}\}$ of $L^2([-\pi,\pi])$.

Which of the following statements are true?

(1) S is bounded

(2) S is closed

(3) S is compact

(4) S is non-compact

Let $f:[0,1]^2 \to \mathbb{R}$ be a function defined by

$f(x,y) = \frac{xy}{x^2+y^2}$ if either $x \neq 0$ or $y \neq 0$

$= 0$ if $x = y = 0$.

Then which of the following statements are true?

(1) f is continuous at $(0,0)$

(2) f is a bounded function

(3) $\int_0^1 \int_0^1 f(x,y)dxdy$ exists

(4) f is continuous at $(1,0)$

Let $U \subseteq \mathbb{R}^n$ be an open subset of $\mathbb{R}^n$ and $f: U \to \mathbb{R}^n$ be a C^∞ −function.
Suppose that for every $x \in U$, the derivative at x, df_x, is non singular. Then which of the following statements are true?

(1) If $V \subset U$ is open then $f(V)$ is open in $\mathbb{R}^n$

(2) $f: U \to f(U)$ is a homeomorphism.

(3) f is one-one

(4) If $V \subset U$ is closed, then $f(V)$ is closed in $\mathbb{R}^n$.

Let (X,d) be a compact metric space. Let $T: X \to X$ be a continuous function satisfying

$\inf_{n\in\mathbb{N}} d(T^n(x), T^n(y)) \neq 0$ for every $x, y \in X$ with $x \neq y$. Then which of the following statements are true?

(1) T is a one-one function

(2) T is not a one-one function

(3) Image of T is closed in X

(4) If X is finite, then T is onto

Let $p(x)$ be a polynomial function in one variable of odd degree and g be a continuous function from $\mathbb{R}$ to $\mathbb{R}$. Then which of the following statements are true.

(1) $\exists$ a point $x_0 \in \mathbb{R}$ such that $p(x_0) = g(x_0)$

(2) If g is a polynomial function then there exists $x_0 \in \mathbb{R}$ such that $p(x_0) = g(x_0)$

(3) If g is a bounded function there exists $x_0 \in \mathbb{R}$ such that $p(x_0) = g(x_0)$

(4) There is a unique point $x_0 \in \mathbb{R}$ such that $p(x_0) = g(x_0)$

Let $n \geq 1$ and $\alpha, \beta \in \mathbb{R}$ with $\alpha \neq \beta$. Suppose $A_n(\alpha, \beta) = [a_{ij}]$ is an $n \times n$ matrix such that $a_{ii} = \alpha$ and $a_{ij} = \beta$ for $i \neq j, 1 \leq i, j \leq n$. Let D_n be the determinant of $A_n(\alpha, \beta)$. Which of the following statements are true?

(1) $D_n = (\alpha - \beta)D_{n-1} + \beta$ for $n \geq 2$

(2) $\frac{D_n}{(\alpha-\beta)^{n-1}} = \frac{D_{n-1}}{(\alpha-\beta)^{n-2}} + \beta$ for $n \geq 2$

(3) $D_n = (\alpha + (n-1)\beta)^{n-1}(\alpha - \beta)$ for $n \geq 2$

(4) $D_n = (\alpha + (n-1)\beta)(\alpha - \beta)^{n-1}$ for $n \geq 2$

Let $T: \mathbb{C}^n \to \mathbb{C}^n$ be a linear transformation, $n \geq 2$. Suppose 1 is the only eigenvalue of T. Which of the following statements are true?

(1) $T^k \neq I$ for any $k \in \mathbb{N}$

(2) $(T - I)^{n-1} = 0$

(3) $(T - I)^n = 0$

(4) $(T - I)^{n+1} = 0$

Let $A \in M_3(\mathbb{R})$ and let $X = \{C \in GL_3(\mathbb{R}) \mid CAC^{-1} \text{is triangular}\}$. Then

(1) $X \neq \emptyset$.

(2) If $X = \emptyset$, then A is not diagonalisable over $\mathbb{C}$

(3) If $X = \emptyset$, then A is diagonalisable over $\mathbb{C}$

(4) If $X = \emptyset$, then A has no real eigenvalue

Let $T: \mathbb{R}^4 \to \mathbb{R}^4$ be a linear transformation with characteristic polynomial $(x-2)^4$ and minimal polynomial $(x-2)^2$. Jordan canonical form of T can be

(1) $\begin{pmatrix} 2 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix}$

(2) $\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix}$

(3) $\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$

(4) $\begin{pmatrix} 2 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$

Let X be a finite dimensional inner product space over $\mathbb{C}$. Let $T: X \to X$ be any linear transformation. Then which of the following statements are true?

(1) T is unitary $\Rightarrow T$ is self adjoint

(2) T is self adjoint $\Rightarrow T$ is normal

(3) T is unitary $\Rightarrow T$ is normal

(4) T is normal $\Rightarrow T$ is unitary

Which of the following statements regarding quadratic forms in 3 variables are true?

(1) Any two quadratic forms of rank 3 are isomorphic over $\mathbb{R}$

(2) Any two quadratic forms of rank 3 are isomorphic over $\mathbb{C}$

(3) There are exactly three non zero quadratic forms of rank ≤ 3 upto isomorphism over $\mathbb{C}$

(4) There are exactly three non zero quadratic forms of rank 2 upto isomorphism over $\mathbb{R}$ and $\mathbb{C}$

Which of the following statements are true?

(1) Any two quadratic forms of same rank in n-variables over $\mathbb{R}$ are isomorphic

(2) Any two quadratic forms of same rank in n-variables over $\mathbb{C}$ are isomorphic

(3) Any two quadratic forms in n-variables are isomorphic over $\mathbb{C}$

(4) A quadratic form in 4 variables may be isomorphic to a quadratic form in 10 variables

Let $U \subset \mathbb{C}$ be an open connected set and $f: U \to \mathbb{C}$ be a non-constant analytic function. Consider the following two sets:

$X = \{z \in U : f(z) = 0\}$

$Y = \{z \in U : f \text{ vanishes on an open neighbourhood of } z \text{ in } U\}$.

Then which of the following statements are true?

(1) X is closed in U

(2) Y is closed in U

(3) X has empty interior

(4) Y is open in U

Consider the power series

$$f(z) = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n)!}$$

Which of the following are true?

(1) Radius of convergence of $f(z)$ is infinite

(2) The set $\{f(x): x \in \mathbb{R}\}$ is bounded

(3) The set $\{f(x): -1 < x < 1\}$ is bounded

(4) $f(z)$ has infinitely many zeroes

Let U be an open subset of $\mathbb{C}$ and $f: U \to \mathbb{C}$ be an analytic function. Then which of the following are true?

(1) If f is one-one, then $f(U)$ is open in $\mathbb{C}$

(2) If f is onto, then $U = \mathbb{C}$.

(3) If f is onto, then f is one-one

(4) If $f(U)$ is closed in $\mathbb{C}$, then $f(U)$ is connected

Let $f: \mathbb{C} \to \mathbb{C}$ be an analytic function. For $z_0 \in \mathbb{C}$, which of the following statements are true?

(1) f can take the value z_0 at finitely many points in $\left\{\frac{1}{n} \mid n \in \mathbb{N}\right\}$

(2) $f(1/n) = z_0$ for all $n \in \mathbb{N} \Rightarrow f$ is the constant function z_0

(3) $f(n) = z_0$ for all $n \in \mathbb{N} \Rightarrow f$ is the constant function z_0

(4) $f(r) = z_0$ for all $r \in \mathbb{Q} \cap [1, 2] \Rightarrow f$ is the constant function z_0

Let $C[0,1]$ be the ring of all real valued continuous function on $[0,1]$.

Let $A = \{f \in C[0,1] : f(^1/_4) = f(^3/_4) = 0\}$. Then which of the following statements are true?

(1) A is an ideal in $C[0,1]$ but is not a prime ideal in $C[0,1]$

(2) A is a prime ideal in $C[0,1]$

(3) A is a maximal ideal in $C[0,1]$

(4) A is a prime ideal in $C[0,1]$, but is not a maximal ideal in $C[0,1]$

For a given integer k, which of the following statements are false?

(1) If $k \pmod{72}$ is a unit in $\mathbb{Z}_{72}$, then $k \pmod 9$ is a unit in $\mathbb{Z}_9$

(2) If $k \pmod{72}$ is a unit in $\mathbb{Z}_{72}$, then $k \pmod 8$ is a unit in $\mathbb{Z}_8$

(3) If $k \pmod 8$ is a unit in $\mathbb{Z}_8$, then $k \pmod{72}$ is a unit in $\mathbb{Z}_{72}$

(4) If $k \pmod 9$ is a unit in $\mathbb{Z}_9$, then $k \pmod{72}$ is a unit in $\mathbb{Z}_{72}$

Let I be an ideal of $\mathbb{Z}$. Then which of the following statements are true?

(1) I is a principal ideal

(2) I is a prime ideal of $\mathbb{Z}$

(3) If I is a prime ideal of $\mathbb{Z}$, then I is a maximal ideal in $\mathbb{Z}$

(4) If I is a maximal ideal in $\mathbb{Z}$, then I is a prime ideal of $\mathbb{Z}$

Let $F[X]$ be the polynomial ring in one variable over a field F. Then which of the following statements are true?

(1) $F[X]$ is a UFD

(2) $F[X]$ is a PID

(3) $F[X]$ is a Euclidean domain

(4) $F[X]$ is a PID but is not an Euclidean domain

Let $f(x) \in \mathbb{Z}[x]$ be a monic polynomial of degree n. Then which of the following are true?

(1) If $f(x)$ is irreducible in $\mathbb{Z}[x]$, then it is irreducible in $\mathbb{Q}[x]$

(2) If $f(x)$ is irreducible in $\mathbb{Q}[x]$, then it is irreducible in $\mathbb{Z}[x]$

(3) If $f(x)$ is reducible in $\mathbb{Z}[x]$, then it has a real root

(4) If $f(x)$ has a real root, then it is reducible in $\mathbb{Z}[x]$

Let F be a field. Then which of the following statements are true?

(1) All extensions of degree 2 of F are isomorphic as fields

(2) All finite extensions of F of same degree are isomorphic as fields if $Char(F) > 0$

(3) All finite extensions of F of same degree are isomorphic as fields if F is finite

(4) All finite normal extensions of F are isomorphic as fields if $Char(F) = 0$

Which of the following statements are true?

(1) There exist three mutually disjoint subsets of $\mathbb{R}$, each of which is countable and dense in $\mathbb{R}$

(2) For each $n \in \mathbb{N}$, there exist n mutually disjoint subsets of $\mathbb{R}$, each of which is countable and dense in $\mathbb{R}$

(3) There exist countably infinite number of mutually disjoint subsets of $\mathbb{R}$, each of which is countable and dense in $\mathbb{R}$

(4) There exist uncountable number of mutually disjoint subsets of $\mathbb{R}$, each of which is countable and dense in $\mathbb{R}$

Consider $[n] = \{1,2,\dots,n\}$ with the discrete topology and let

$$X = \prod_{n\geq 1} [n]$$

be the product space with the product topology. For $x = (a_1, a_2, \dots) \in X$, define $T(x) = (1, a_1, a_2, \dots)$. Then which of the following statements are true?

(1) Let $x_n \in X$ for $n = 1,2,3,\dots$ be a sequence in X. Then it is convergent.

(2) X is a compact, Hausdorff space

(3) The map $T: X \to X$ is continuous

(4) The map $T: X \to X$ has a unique fixed point

The integral equation

$$\phi(x) = 1 + \frac{2}{\pi}\int_0^{\pi} (\cos^2 x)\,\phi(t)\,dt$$

has

(1) no solution

(2) unique solution

(3) more than one but finitely many solutions

(4) infinitely many solutions

Consider the initial value problem $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$; $0 \leq x \leq 1$. Then which of the following statements are true?

(1) There exists a unique solution in $\left[0, \frac{\pi}{4}\right]$

(2) Every solution is bounded in $\left[0, \frac{\pi}{4}\right]$

(3) The solution exhibits a singularity at some point in $[0,1]$

(4) The solution becomes unbounded in some subinterval of $\left[\frac{\pi}{4}, 1\right]$

Consider the eigenvalue problem

$((1+x^4)y')' + \lambda y = 0, x \in (0,1),$

$y(0) = 0, y(1) + 2y'(1) = 0.$

Then which of the following statements are true?

(1) all the eigenvalues are negative

(2) all the eigenvalues are positive

(3) there exist some negative eigenvalues and some positive eigenvalues

(4) there are no eigenvalues

Let y be a solution of

$(1+x^2)y'' + (1+4x^2)y = 0, x > 0$

$y(0) = 0$. Then y has

(1) infinitely many zeros in $[0,1]$

(2) infinitely many zeros in $[1,\infty)$

(3) at least n zeros in $[0, n\pi], \forall n \in \mathbb{N}$

(4) at most $3n$ zeros in $[0, n\pi], \forall n \in \mathbb{N}$

A possible initial strip $(x_0, y_0, z_0, p_0, q_0)$ for the Cauchy problem $pq = 1$

where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$ and $x_0(s) = s, y_0(s) = \frac{1}{s}, z_0(s) = 1$ for $s > 1$ is

(1) $\left(s, \frac{1}{s}, 1, \frac{1}{s}, s\right)$

(2) $\left(s, \frac{1}{s}, 1, -\frac{1}{s}, -s\right)$

(3) $\left(s, \frac{1}{s}, 1, \frac{1}{s}, -s\right)$

(4) $\left(s, \frac{1}{s}, 1, -\frac{1}{s}, s\right)$

Let $u(x, t)$ be the solution of

$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = xt$, $-\infty < x < \infty, t > 0,$

$u(x, 0) = \frac{\partial u}{\partial t}(x, 0) = 0, -\infty < x < \infty.$

Then $u(2,3)$ is equal to

(1) 9

(2) 1

(3) 27

(4) 12

The values of α, A, B, C for which the quadrature formula

$$\int_{-1}^{1}(1-x)f(x)dx = Af(-\alpha) + Bf(0) + Cf(\alpha)$$

is exact for polynomials of highest possible degree, are

(1) $\alpha = \sqrt{\frac{3}{5}}, A = \frac{5}{9} + \frac{\sqrt{5}}{3\sqrt{3}}, B = \frac{8}{9}, C = \frac{5}{9} - \frac{\sqrt{5}}{3\sqrt{3}}$

(2) $\alpha = \sqrt{\frac{3}{5}}, A = \frac{5}{9} - \frac{\sqrt{5}}{3\sqrt{3}}, B = \frac{8}{9}, C = \frac{5}{9} + \frac{\sqrt{5}}{3\sqrt{3}}$

(3) $\alpha = \sqrt{\frac{3}{5}}, A = \frac{5}{9}\left(1 - \frac{\sqrt{3}}{\sqrt{5}}\right), B = \frac{8}{9}, C = \frac{5}{9}\left(1 + \frac{\sqrt{3}}{\sqrt{5}}\right)$

(4) $\alpha = \sqrt{\frac{3}{5}}, A = \frac{5}{9}\left(1 + \frac{\sqrt{3}}{\sqrt{5}}\right), B = \frac{8}{9}, C = \frac{5}{9}\left(1 - \frac{\sqrt{3}}{\sqrt{5}}\right)$

Consider the ordinary differential equation (ODE)

$$\begin{cases} y'(x) + y(x) = 0, & x > 0, \\ y(0) = 1, \end{cases}$$

and the following numerical scheme to solve the ODE

$$\begin{cases} \frac{Y_{n+1} - Y_{n-1}}{2h} + Y_{n-1} = 0, & n \geq 1, \\ Y_0 = 1, Y_1 = 1. \end{cases}$$

If $0 < h < \frac{1}{2}$, then which of the following statements are true?

(1) $(Y_n) \to \infty$ as $n \to \infty$

(2) $(Y_n) \to 0$ as $n \to \infty$

(3) (Y_n) is bounded

(4) $\max_{nh \in [0,T]} |y(nh) - Y_n| \to \infty$ as $T \to \infty$

The minimum value of the functional

$I(y) = \int_0^{\pi} \left(\frac{dy}{dx}\right)^2 dx,$

subject to $\int_0^{\pi} y^2(x)dx = 1,\ y(0) = 0 = y(\pi)$ is equal to

(1) 1/2

(2) 1

(3) 2

(4) 1/3

Let $y = y(x) \in C^4([0,1])$ be an extremizing function for the functional

$I(y) = \int_0^1 \left[\left(\frac{d^2y}{dx^2}\right)^2 - 2y\right] dx,$ satisfying $y(0) = 0 = y(1)$. Then an extremal $y(x)$, satisfying the given conditions at 0 and 1 together with the natural boundary conditions, is given by

(1) $\frac{x}{24}(x-1)^3$

(2) $\frac{x^2}{24}(x-1)^2$

(3) $\frac{x}{24}(x^3 - 2x^2 + 1)$

(4) $\frac{x}{24}(x^3 + x^2 - 2)$

Assume that h_1, h_2, g_1 and $g_2 \in C([a,b])$.

Let $\phi(x) = f(x) + \lambda \int_a^b [h_1(t)g_1(x) + h_2(t)g_2(x)]\phi(t)dt$

be an integral equation. Consider the following statements:

S_1: If the given integral equation has a solution for some $f \in C([a,b])$, then

$\int_a^b f(t)g_1(t)dt = 0 = \int_a^b f(t)g_2(t)dt.$

S_2: The given integral equation has a unique solution for every $f \in C([a,b])$ if λ is not a characteristic number of the corresponding homogeneous equation.

Then

(1) Both S_1 and S_2 are true

(2) S_1 is true but S_2 is false

(3) S_1 is false but S_2 is true

(4) Both S_1 and S_2 are false

Consider a mechanical system whose position is described using the generalized coordinates $q_1, \ldots, q_n$. Let $T(q_1, \ldots, q_n, \dot{q}_1, \ldots, \dot{q}_n)$ be the kinetic energy of the system. If the generalized force Q_j, $1 \le j \le n$, acting on the system is zero, then the Lagange equations of motion are

(1) $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = 0,\ 1 \le j \le n$

(2) $\frac{d}{dt}\left(\frac{\partial T}{\partial q_j}\right) - \frac{\partial T}{\partial \dot{q}_j} = 0,\ 1 \le j \le n$

(3) $\frac{\partial}{\partial \dot{q}_j}\left(\frac{dT}{dt}\right) - 2\frac{\partial T}{\partial q_j} = 0,\ 1 \le j \le n$

(4) $\frac{\partial}{\partial \dot{q}_j}\left(\frac{dT}{dt}\right) - \frac{\partial T}{\partial q_j} = 0,\ 1 \le j \le n$

Let X and Y be real-valued independent random variables on Ω. Then which of the following statements are correct?

(1) $E[\cos(tX + uY)] = E[\cos(tX)]E[\cos(uY)] - E[\sin(tX)]\,E[\sin(uY)]$ for all $t, u \in \mathbb{R}$

(2) If $X \sim N(2,1)$ and $Y \sim N(0,2)$, then $\mathrm{Var}(X+Y) = 3$ where $N(\mu, \sigma^2)$ represents Normal distribution with mean μ and variance σ^2

(3) $\{X = a\} \cap \{Y = b\} = \phi$ for all $a, b \in \mathbb{R}$

(4) $P(\{X = a\} \cap \{Y = b\}) = P(\{X = a\})P(\{Y = b\})$ for all $a, b \in \mathbb{R}$

A random variable T has a symmetric distribution if T and $-T$ have the same distribution. Let X and Y be independent random variables. Then which of the following statements are correct?

(1) If X and Y have the same distribution then $X - Y$ has a symmetric distribution

(2) If $X \sim N(3,1)$ and $Y \sim N(2,2)$, then $2X - 3Y$ has a symmetric distribution

(3) If X and Y have the same symmetric distribution, then $X + Y$ has a symmetric distribution

(4) If X has a symmetric distribution, then XY has a symmetric distribution

Let $\{(X_n, Y_n): n \geq 1\}$ and (X, Y) be random variables, on $(\Omega, \mathcal{F}, P)$. Then which of the following statements are correct?

(1) If $X_n \to X$ almost surely, $Y_n \to Y$ almost surely, then $X_n + Y_n \to X + Y$ in distribution

(2) If $X_n \to X$ in probability, $Y_n \to Y$ almost surely, then $X_n + Y_n \to X + Y$ in distribution

(3) If $X_n \to X$ in probability, $Y_n \to Y$ in probability, then $X_n + Y_n \to X + Y$ in distribution

(4) If $X_n \to X$ in distribution, $Y_n \to Y$ in distribution, then $X_n + Y_n \to X + Y$ in distribution

Let $\{X_n : n \geq 0\}$ be a Markov chain with state space $\mathbb{N} \cup \{0\}$ such that the transition probabilities are given by

$$p_{ij} = \begin{cases} q & \text{for } j = 0 \\ 1-q & \text{for } j = i+1 \\ 0 & \text{otherwise} \end{cases}$$

for $= 0,1,2,\dots$, where $0 < q < 1$. Then which of the following statements are correct?

(1) The Markov chain is irreducible

(2) The Markov chain is aperiodic

(3) $p_{00}^{(n)} = q$ for all $n \geq 1$

(4) The Markov chain is positive recurrent

Consider a Markov chain with state space S. Let $d(k)$ denote the period of state $k \in S$. Which of the following statements are correct?

(1) For $i, j \in S$, if $\exists\, n, m > 0$ such that $p_{ij}^{(n)} > 0$ and $p_{ji}^{(m)} > 0$ and i is recurrent, then j is recurrent

(2) For $i, j \in S$, if $\exists\, n, m > 0$ such that $p_{ij}^{(n)} > 0$ and $p_{ji}^{(m)} > 0$, then $d(i) = d(j)$

(3) For $i, j \in S$, if $\exists\, r > 0$ such that $p_{ij}^{(r)} > 0$ then j cannot be transient

(4) For $i, j \in S$, if $\exists\, r > 0$ such that $p_{ij}^{(r)} > 0$ and i is null recurrent then j is positive recurrent

Suppose $X_1, X_2, \dots, X_n$ are i.i.d. Uniform $(\theta, 2\theta), \theta > 0$. Let $X_{(1)} = \min\{X_1, \dots, X_n\}$ and $X_{(n)} = \max\{X_1, \dots, X_n\}$. Then which of the following statements are correct?

(1) $(X_{(1)}, X_{(n)})$ is jointly sufficient and complete for θ

(2) $(X_{(1)}, X_{(n)})$ is jointly sufficient but not complete for θ

(3) $\frac{X_{(n)}}{2}$ is a maximum likelihood estimator for θ

(4) $X_{(1)}$ is a maximum likelihood estimator for θ

Let $\{X_i : i \geq 1\}$ be i.i.d. observations with $E(X_i) = 0$ and Var $(X_i) = \sigma^2 > 0$. Then which of the following statements are correct?

(1) $\frac{1}{n}\sum_{i=1}^{n} (X_{2i-1} - X_{2i})^2$ is consistent for σ^2

(2) $\frac{1}{n}\sum_{i=1}^{n} (X_i - \bar{X})^2$ is consistent for σ^2

(3) $\frac{1}{n}\sum_{i=1}^{n} X_i^2$ is consistent for σ^2

(4) $\frac{1}{n-1}\sum_{i=1}^{n} (X_i - \bar{X})^2$ is consistent for σ^2

Let X be a discrete random variable with sample space $\mathcal{X} = \{1, 2, \dots, 10\}$ and probability mass function $p(x), x \in \mathcal{X}$. Consider testing the hypothesis

$H_0 : p(x) = \frac{1}{10}, x \in \mathcal{X}$ against

$H_1 : p(x) \propto x, \ x \in \mathcal{X}$

based on a single observation X. Then which of the following statements are correct?

(1) The test with critical region $\{X \geq 2\}$ is most powerful of its size

(2) The test with critical region $\{X < 2\}$ is unbiased at level $\alpha = 0.1$

(3) If $X = 7$ the p-value of the most powerful test is 0.6

(4) There exists a nonrandomized test of size 0.05

Let $\{X_n : n \geq 1\}$ be i.i.d. with common unknown continuous distribution function $F(x-\theta)$, where θ is the unique median of F. Define

$$Y_i = \begin{cases} 1 & \text{if } X_i > 1 \\ 0 & \text{if } X_i \leq 1 \end{cases} \quad \text{and } S_n = \sum_{i=1}^{n} Y_i .$$

For testing $H_0 : \theta = 1$ against $H_1 : \theta > 1$, which of the following statements are correct?

(1) $S_n \sim$ Binomial $\left(n, \frac{1}{2}\right)$ under H_0

(2) Test based on S_n is distribution-free under H_1

(3) Right-tailed test based on S_n is unbiased

(4) The sequence of right-tailed tests based on $S_n, n \geq 1$, is consistent

Suppose the conditional p.d.f. of a random variable X given θ is

$$f(x|\theta) = \begin{cases} \frac{2x}{\theta^2}, & 0 < x < \theta \\ 0, & \text{otherwise} \end{cases}$$

where the prior distribution of θ is Uniform (0,1). Based on a single observation x from X, which of the following statements are correct?

(1) The Bayes estimate for θ under squared error loss function is $-x \log_e x$

(2) The Bayes estimate for θ under squared error loss function is $-\frac{x \log_e x}{1-x}$

(3) The Bayes estimate for θ under absolute error loss function is x

(4) The Bayes estimate for θ under absolute error loss function is $\frac{2x}{1+x}$

Consider a Balanced Incomplete Block Design (v, b, r, k, λ) of v treatments and b blocks of k plots each. Let N be the $v \times b$ incidence matrix of the design. Then which of the following statements are correct?

(1) $\lambda(k-1) = r(v-1)$

(2) $b \geq v$

(3) rank $(NN') = v$

(4) trace $(NN') = bk$

Consider the random effect model $y_{ij} = \mu + b_i + \varepsilon_{ij}, i = 1, \dots, 5; j = 1, \cdots, 10$ where $b_i \sim \text{i. i. d.}\, N(0, \tau^2)$ and $\varepsilon_{ij} \sim \text{i. i. d.}\, N(0, \sigma^2)$ are all independent of each other. The parameter space for the model is $(\mu, \sigma^2, \tau^2) \in \mathbb{R} \times [0, \infty) \times [0, \infty)$. Let $\hat{\sigma}_u^2$ and $\hat{\tau}_u^2$ be the usual unbiased ANOVA estimators of σ^2 and τ^2 respectively, and $\hat{\sigma}_m^2$ and $\hat{\tau}_m^2$ be the maximum likelihood estimators of σ^2 and τ^2 respectively. Then, which of the following events can happen with positive probability for some parameter values?

(1) $\hat{\sigma}_u^2$ is negative

(2) $\hat{\tau}_u^2$ is negative

(3) $\hat{\sigma}_m^2$ is negative

(4) $\hat{\tau}_m^2$ is negative

Given data $\{(x_i, y_i) : i = 1, 2, \dots, n\}$ where $n \geq 2$ and not all x_i's are identical, the simple linear regression model $y = \alpha + \beta x + \varepsilon$ is fit. Let h_{ii} be the i^{th} diagonal element of the Hat matrix $H = X(X'X)^{-1}X'$, where $X_{n\times 2}$ is the correspondingmodel matrix. Then which of the following are possible for some choice of n and $x_1, x_2, \dots, x_n$?

(1) $h_{ii} = -1$ for some i

(2) $h_{ii} = 0$ for some i

(3) $h_{ii} = 1$ for some i

(4) All h_{ii} are equal

Suppose that $M_{p\times p} \sim \text{Wishart}_p(I, m)$ and $Z \sim N_p(\mathbf{0}, \Sigma)$ are independent where Σ is positive definite. Let $a \in \mathbb{R}^p$ be a fixed p-vector such that $a \neq \mathbf{0}$ and define $d = Z/\|Z\|$. Then which of the following random variables has a χ^2 distribution, possibly after being scaled by a constant factor?

(1) $d' M^{-1} d$

(2) $(d' M^{-1} d)^{-1}$

(3) $a' M a$

(4) $(a' M^{-1} a)^{-1}$

To estimate the population total $Y = \sum_{i=1}^{N} y_i$, where $y_1, y_2, \dots, y_N$ are study variables and N is the size of the finite population, a sample of size n is drawn using $PPSWR(p_1, p_2, \dots, p_N)$ scheme. If $\hat{Y}_{HH}$ is the Hansen-Hurwitz estimator for Y then which of the following are correct?

(1) $\hat{Y}_{HH}$ is unbiased for Y

(2) $\text{Var}(\hat{Y}_{HH}) = \frac{1}{n}\left\{\sum_{i=1}^{N} \frac{y_i^2}{p_i} - Y^2\right\}$

(3) $\text{Var}(\hat{Y}_{HH}) = \frac{1}{n}\sum_{i=1}^{N}\left\{\frac{y_i}{p_i} - E(\hat{Y}_{HH})\right\}^2 p_i$

(4) $\text{Var}(\hat{Y}_{HH}) = \frac{1}{n(n-1)}\sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N}\left(\frac{y_i}{p_i} - \frac{y_j}{p_j}\right)^2 p_i p_j$

Consider the following system with three independent components u_1, u_2 and u_3:

Suppose that the failure probability of each component is p, and let $f(p)$ be the probability that the whole system is still functioning. Then which of the following statements are correct?

(1) $f\left(\frac{1}{2}\right) = \frac{5}{8}$

(2) $f\left(\frac{1}{2}\right) = \frac{3}{8}$

(3) $f\left(\frac{1}{3}\right) = \frac{22}{27}$

(4) $f\left(\frac{1}{4}\right) = \frac{50}{64}$

Suppose in a single service queue, customers arrive at a Poisson rate of one per ten minutes, and the service time is Exponential at a rate of one service per five minutes. Let P_n be the probability that there are n customers in the system in steady state. Then which of the following statements are correct?

(1) $P_{n+1} = \frac{1}{2} P_n$ for all $n \geq 0$

(2) The expected number of customers in the system is 1 in steady state

(3) The expected number of customers in the system is 2 in steady state

(4) The expected amount of time a customer spends in the system in steady state is 10 minutes

Let X_1, X_2 and X_3 be i.i.d. Normal random variables with mean θ and variance θ^2 where $\theta \in \mathbb{R}$ is unknown. Then which of the following statements are correct?

(1) $\frac{X_1 + 2X_2 + 3X_3}{6}$ is unbiased for θ

(2) $\frac{X_1^2 + 4X_2^2 + 9X_3^2}{14}$ is unbiased for θ^2

(3) $\frac{2X_1 + X_2^2}{2}$ is unbiased for $\theta(1 + \theta)$

(4) $X_2\left(1 - \frac{X_2}{2}\right)$ is unbiased for $\theta(1 - \theta)$

Exam_Date:27.12.2019

National Testing Agency

Subject:MATHEMATICAL SCIENCES

JOINT CSIR -UGC NET DEC. 2019 Final Answer Key on which score compiled and declared

SNO	QUESTION ID	CORRECT OPTION(S)
001	3398932058	3398938003
002	3398932059	3398938007
003	3398932060	3398938009
004	3398932061	3398938013
005	3398932062	3398938019
006	3398932063	3398938021
007	3398932064	3398938025
008	3398932065	3398938030
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010	3398932067	3398938038
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013	3398932070	3398938050
014	3398932071	3398938055
015	3398932072	3398938058
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022	3398932079	3398938085
023	3398932080	3398938091
024	3398932081	3398938096
025	3398932082	3398938100
026	3398932083	3398938102
027	3398932084	3398938106
028	3398932085	3398938111
029	3398932086	3398938115
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039	3398932096	3398938155
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041	3398932098	3398938161
042	3398932099	3398938167
043	3398932100	3398938171
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069	3398932126	3398938275
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079	3398932136	3398938315
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084	3398932141	3398938333 , 3398938334
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089	3398932146	3398938355 , 3398938356
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091	3398932148	3398938361 , 3398938362 , 3398938364
092	3398932149	3398938365 , 3398938366 , 3398938368
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097	3398932154	3398938385 , 3398938387
098	3398932155	3398938391
099	3398932156	3398938393 , 3398938396
100	3398932157	3398938397 , 3398938398 , 3398938400
101	3398932158	3398938401 , 3398938402 , 3398938403
102	3398932159	3398938406 , 3398938407
103	3398932160	3398938410 , 3398938411 , 3398938412
104	3398932161	3398938414 , 3398938415
105	3398932162	3398938418 , 3398938419 , 3398938420
106	3398932163	3398938423
107	3398932164	3398938426 , 3398938427
108	3398932165	3398938429 , 3398938431
109	3398932166	3398938434
110	3398932167	3398938437 , 3398938439
111	3398932168	3398938441 , 3398938442 , 3398938443
112	3398932169	3398938445 , 3398938446 , 3398938447
113	3398932170	3398938449 , 3398938451
114	3398932171	3398938454 , 3398938456
115	3398932172	3398938458 , 3398938460
116	3398932173	3398938461 , 3398938462 , 3398938463 , 3398938464
117	3398932174	3398938465 , 3398938468
118	3398932175	3398938471 , 3398938472
119	3398932176	3398938473
120	3398932177	3398938477 , 3398938480

NB:Correct Option ID '..............' means the question cancelled.

15.01.2020