

CSIR 27th Dec 2019 S2

Application No.	
Candidate Name	
Roll No.	
Test Date	27/12/2019
Test Time	2:30 PM - 5:30 PM
Subject	Mathematical Sciences

Section : PART-A General Aptitude

Q.1 Water drips out of the bottom of a cylindrical bucket that is initially full. The rate of dripping is proportional to the height of water column in the bucket. If the rate of dripping at half height is R , then the average rate of dripping until the bucket becomes almost empty, is

1. greater than R
2. R
3. between $R/2$ and R
4. less than $R/2$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ

Question ID : 3398932065

Option 1 ID : 3398938029

Option 2 ID : 3398938030

Option 3 ID : 3398938031

Option 4 ID : 3398938032

Status : Not Answered

Chosen Option : --

Q.2 A unicellular organism reproduces by cell division. When two of the organisms come together they tend to destroy each other. If n is the number of cells, which of the following equations best represents the rate of change of the population [where α and β are positive constants]?

1. $\frac{dn}{dt} = \alpha n - \beta n^2$
2. $\frac{dn}{dt} = -\alpha n - \beta n^2$
3. $\frac{dn}{dt} = \alpha n + \beta n^2$
4. $\frac{dn}{dt} = -\alpha n + \beta n^2$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ

Question ID : 3398932061

Option 1 ID : 3398938013

Option 2 ID : 3398938014

Option 3 ID : 3398938015

Option 4 ID : 3398938016

Q.3 Governor : State ::

1. ship : captain
2. admiral : navy
3. jail : prisoner
4. student : teacher

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ

Question ID : 3398932073

Option 1 ID : 3398938061

Option 2 ID : 3398938062

Option 3 ID : 3398938063

Option 4 ID : 3398938064

Status : Not Answered

Chosen Option : --

Q.4 Person X purchases a house at a price P and sells it to Y at a profit of 10%. Y sells it back to X incurring a loss of 10%. As a result

1. X makes a profit of 11 % of P
2. Y incurs a loss of 10% of P
3. Y makes a profit of 11 % of P
4. Neither X nor Y gains or loses

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ

Question ID : 3398932060

Option 1 ID : 3398938009

Option 2 ID : 3398938010

Option 3 ID : 3398938011

Option 4 ID : 3398938012

Status : Not Answered

Chosen Option : --

Q.5 Three metal cubes whose diagonals are $\sqrt{3}$, $6\sqrt{3}$ and $8\sqrt{3}$ units respectively, are melted to make a new cube. The side of the new cube would be

1. 15 units
2. $10\sqrt{3}$ units
3. $15\sqrt{3}$ units
4. 9 units

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCQ

Question ID : 3398932069

Option 1 ID : 3398938045

Option 2 ID : 3398938046

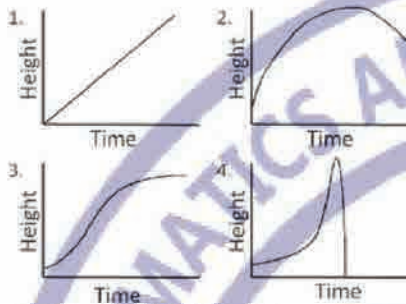
Option 3 ID : 3398938047

Option 4 ID : 3398938048

Status : Marked For Review

Chosen Option : 4

Q.6 Which of the following graphs correctly shows the growth of a sapling to a mature tree?



- Options
1. 1
 2. 2
 3. 3
 4. 4

Question Type : MCQ

Question ID : 3398932066

Option 1 ID : 3398938033

Option 2 ID : 3398938034

Option 3 ID : 3398938035

Option 4 ID : 3398938036

Status : Marked For Review

Chosen Option : 3

Q.7 The following are the three positions of a die with numbers 1 to 6 written on its faces. Then the number 6 appears on the face opposite that of



1. 1
2. 2
3. 4
4. 5

- Options
1. 1
 2. 2
 3. 3
 4. 4

Question Type : MCQ

Question ID : 3398932070

Option 1 ID : 3398938049

Option 2 ID : 3398938050

Option 3 ID : 3398938051

Q.8 In a group of researchers Biologists are thrice as many as Chemists whereas the Physicists are twice as many as Biologists. If there are 5 Chemists in the group, what is the total number of these three types of researchers?

1. 30
2. 45
3. 50
4. 75

Options 1. 1

2. 2
3. 3
4. 4

Question Type : MCO

Question ID : 3398932059

Option 1 ID : 3398938005

Option 2 ID : 3398938006

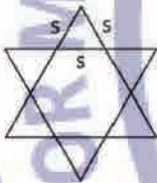
Option 3 ID : 3398938007

Option 4 ID : 3398938008

Status : Marked For Review

Chosen Option : 3

Q.9 What is the area of the figure shown below, assuming all small triangles to be equilateral triangles of side S ?



1. $3\sqrt{3}S^2$
2. $6S^2$
3. $4\sqrt{3}S^2$
4. $3S^2$

Options 1. 1

2. 2
3. 3
4. 4

Question Type : MCO

Question ID : 3398932068

Option 1 ID : 3398938041

Option 2 ID : 3398938042

Option 3 ID : 3398938043

Option 4 ID : 3398938044

Status : Not Answered

Chosen Option : --

Q.10

“The father of my biological son is the only child of your parents.”

The statement can be true

1. under no condition
2. only if a woman says so
3. only if a married man says so
4. only if an unmarried man says so

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932075

Option 1 ID : 3398938069

Option 2 ID : 3398938070

Option 3 ID : 3398938071

Option 4 ID : 3398938072

Status : Not Answered

Chosen Option : --

Q.11 The product of the ten numbers 10, 11, 12, 19, 20, 23, 25, 33, 35, and 50 is divisible by 10^x . What is the largest possible value of x ?

1. 4

2. 5

3. 6

4. 7

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932071

Option 1 ID : 3398938053

Option 2 ID : 3398938054

Option 3 ID : 3398938055

Option 4 ID : 3398938056

Status : Marked For Review

Chosen Option : 3

Q.12 The straight line $y_1 = 4$ and the curve $y_2 = \sin(4x)$, with x varying from 0 to 450,

1. do not intersect at all.
2. touch each other at only one point.
3. intersect at 5 points.
4. intersect at 10 points.

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932064

Option 1 ID : 3398938025

Option 2 ID : 3398938026

Option 3 ID : 3398938027

Option 4 ID : 3398938028

Status : Not Answered

Chosen Option : --

Q.13 In an examination, if more than 15 questions are attempted, only the first 15 attempted ones are evaluated. Answers are awarded + 2 marks if correct and – 0.5 if wrong. A candidate answers 19 questions and gets 15 marks. How many questions are answered correctly in the first fifteen attempted?

1. 9
2. 10
3. 11
4. 12

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932063

Option 1 ID : 3398938021

Option 2 ID : 3398938022

Option 3 ID : 3398938023

Option 4 ID : 3398938024

Status : Marked For Review

Chosen Option : 1

Q.14 Birds beat their wings up and down but propel themselves forward because

1. the neck is held at an angle with the body axis
2. the flight feathers make an angle with the wing axis
3. their body is streamlined
4. their tail is fan shaped

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932074

Option 1 ID : 3398938065

Option 2 ID : 3398938066

Option 3 ID : 3398938067

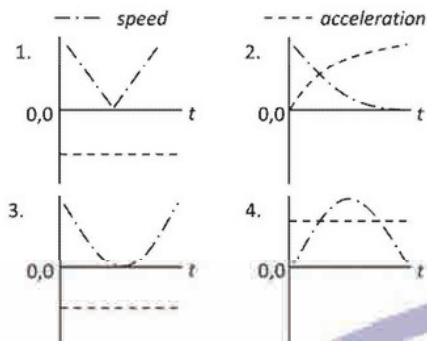
Option 4 ID : 3398938068

Status : Not Answered

Chosen Option : --

Q.15

Which of the following graphs shows the speed and the acceleration of a ball thrown up from the Earth and falling back vertically?



Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932076

Option 1 ID : 3398938073

Option 2 ID : 3398938074

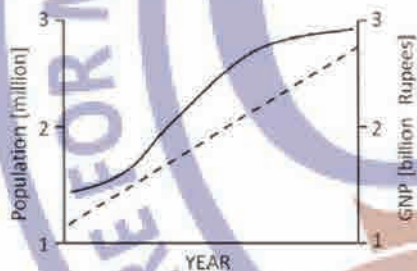
Option 3 ID : 3398938075

Option 4 ID : 3398938076

Status : Not Answered

Chosen Option : --

Q.16 The graph shows the population (*solid line*) and the Gross National Product (GNP) (*dash line*) of a country over several years.



Which of the following statements can be inferred from the graph?

1. The rate of change is the same for both, the population and the GNP
2. The population has grown faster than the GNP over the entire period
3. Per capita income at the end of the period is greater than at the beginning
4. Per capita income increased because of growth in the population

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932058

Option 1 ID : 3398938001

Option 2 ID : 3398938002

Option 3 ID : 3398938003

Option 4 ID : 3398938004

Status : Not Answered

Chosen Option : --

Q.17 An LED bulb costs 5 times as much as a filament bulb but consumes only $\frac{1}{5}$ of electrical power. If C is the cost of the filament bulb and P is the electric power tariff, the extra cost of the LED bulb will be recovered (in comparison to the filament bulb) in usage time of

1. 5 years
2. $5C/P$
3. $C/5P$
4. C/P

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCO

Question ID : 3398932067

Option 1 ID : 3398938037

Option 2 ID : 3398938038

Option 3 ID : 3398938039

Option 4 ID : 3398938040

Status : Not Answered

Chosen Option : --

Q.18 An ant finds a sugar heap having 1000 grains. It takes one grain back to the anthill and then takes a friend along, to fetch one grain back each. Ants repeat the trips, each ant adding a new friend at every trip, until the heap is exhausted. The number of ants that went out is

1. 256
2. 450
3. 512
4. 1000

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCO

Question ID : 3398932062

Option 1 ID : 3398938017

Option 2 ID : 3398938018

Option 3 ID : 3398938019

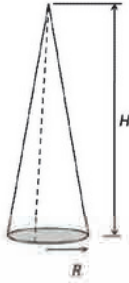
Option 4 ID : 3398938020

Status : Not Answered

Chosen Option : --

Q.19

A hollow paper cone of height H and radius R is cut along the dotted line and opened to form a sector of a circle. The angle subtended by the sector (in radians) is



1. $2\pi \frac{R}{H}$
2. $2\pi \frac{H}{\sqrt{R^2+H^2}}$
3. $2\pi \frac{R}{\sqrt{R^2+H^2}}$
4. $2\pi \frac{H}{R}$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932077

Option 1 ID : 3398938077

Option 2 ID : 3398938078

Option 3 ID : 3398938079

Option 4 ID : 3398938080

Status : Not Answered

Chosen Option : --

Q.20 Among the following pairs of numbers, the smallest difference is between

1. 1.0×10^{23} and 1.0×10^{-23}
2. 1.0×10^{-23} and -1.0×10^{-23}
3. 1.0×10^{23} and -1.0×10^{23}
4. 1.0×10^{23} and -1.0×10^{-23}

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932072

Option 1 ID : 3398938057

Option 2 ID : 3398938058

Option 3 ID : 3398938059

Option 4 ID : 3398938060

Status : Marked For Review

Chosen Option : 2

Let $A = \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$ be a matrix. Which of the following is true?

1. $A^* = A^{-1}$
2. $AA^* = A^*A$
3. $A^* = A$
4. $A^2 = Id$ (2×2 identity matrix)

Options

1. 1
2. 2
3. 3
4. 4

Question Type : MCQ

Question ID : 3398932087

Option 1 ID : 3398938117

Option 2 ID : 3398938118

Option 3 ID : 3398938119

Option 4 ID : 3398938120

Status : Marked For Review

Chosen Option : 1

Q.2 Let A and B be invertible $n \times n$ matrices with entries in $\mathbb{R}$ such that AB is diagonalizable with eigenvalues $\lambda_1, \dots, \lambda_n$. Which of the following is NOT always true?

1. BA is invertible
2. BA is diagonalizable
3. $BA = AB$
4. Eigenvalues of BA are $\lambda_1, \dots, \lambda_n$

Options

1. 1
2. 2
3. 3
4. 4

Question Type : MCQ

Question ID : 3398932086

Option 1 ID : 3398938113

Option 2 ID : 3398938114

Option 3 ID : 3398938115

Option 4 ID : 3398938116

Status : Not Answered

Chosen Option : --

Q.3

Let $\alpha > 0$ be a real number. Then

$$\lim_{n \rightarrow \infty} \frac{1}{n^{\alpha+1}} (1^\alpha + 2^\alpha + \dots + n^\alpha) \text{ is}$$

1. ∞
2. equal to 0
3. equal to 1
4. positive and strictly less than 1

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932082

Option 1 ID : 3398938097

Option 2 ID : 3398938098

Option 3 ID : 3398938099

Option 4 ID : 3398938100

Status : Marked For Review

Chosen Option : 4

Q.4

Which of the following is the Jordan canonical form of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ over $\mathbb{R}$?

1. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$
2. $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$
3. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
4. $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932088

Option 1 ID : 3398938121

Option 2 ID : 3398938122

Option 3 ID : 3398938123

Option 4 ID : 3398938124

Status : Marked For Review

Chosen Option : 1

Q.5

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a continuous function and $(x_n)_{n \geq 1}$ be a bounded sequence of real numbers. Then which of the following is true?

1. $\liminf_{n \rightarrow \infty} f(x_n) = f\left(\liminf_{n \rightarrow \infty} x_n\right)$
2. $\limsup_{n \rightarrow \infty} f(x_n) = f\left(\limsup_{n \rightarrow \infty} x_n\right)$
3. $\liminf_{n \rightarrow \infty} f(x_n) \leq \limsup_{n \rightarrow \infty} x_n$
4. $\liminf_{n \rightarrow \infty} f(x_n) \leq f\left(\liminf_{n \rightarrow \infty} x_n\right)$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCO

Question ID : 3398932081

Option 1 ID : 3398938093

Option 2 ID : 3398938094

Option 3 ID : 3398938095

Option 4 ID : 3398938096

Status : Not Answered

Chosen Option : --

Q.6 Let $f: X \rightarrow Y$ be a function and $(A_n)_{n \geq 1}$ be a sequence of subsets of X . For $A \subset X, A \neq \emptyset$, let $f(A) = \{f(a): a \in A\}$ and $f(\emptyset) = \emptyset$. Then which of the following is true?

1. $f\left(\bigcap_{n \geq 1} A_n\right) = \bigcap_{n \geq 1} f(A_n)$
2. $f\left(\bigcap_{n \geq 1} A_n\right)$ is a proper subset of $\bigcap_{n \geq 1} f(A_n)$
3. $f\left(\bigcup_{n \geq 1} A_n\right) = \bigcup_{n \geq 1} f(A_n)$
4. For any nonempty proper subset A of X , $f(A^c) = f(A)^c$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCO

Question ID : 3398932080

Option 1 ID : 3398938089

Option 2 ID : 3398938090

Option 3 ID : 3398938091

Option 4 ID : 3398938092

Status : Not Answered

Chosen Option : --

Q.7

Let $x > 100$ be a given real number, which is not an integer. Let S be the set of all rational numbers $r \leq x$ of the form

$$r = [x] + \frac{a_1}{10} + \frac{a_2}{10^2} + \dots + \frac{a_n}{10^n}$$

for some natural number $n \geq 1$, where $a_1, \dots, a_n$ are integers such that $0 \leq a_i \leq 9$ and $[x]$ denotes the largest integer $\leq x$. Then which one of the following is true?

1. S is infinite and the supremum of S is x
2. S is infinite and the supremum of S is an integer
3. S is finite and the supremum of S is x
4. S is finite and the supremum of S is an integer

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932079

Option 1 ID : 3398938085

Option 2 ID : 3398938086

Option 3 ID : 3398938087

Option 4 ID : 3398938088

Status : Not Answered

Chosen Option : --

Q.8 Consider a quadratic form $Q(x, y, z)$ on $\mathbb{R}^3$. Let $E_Q = \{(x, y, z) : Q(x, y, z) = 1\}$. For which of the following is E_Q a bounded subset of $\mathbb{R}^3$?

1. $Q(x, y, z) = x^2 + y^2 + z^2$
2. $Q(x, y, z) = x^2 + y^2 - z^2$
3. $Q(x, y, z) = x^2 - y^2 - z^2$
4. $Q(x, y, z) = -x^2 + y^2 - z^2$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932089

Option 1 ID : 3398938125

Option 2 ID : 3398938126

Option 3 ID : 3398938127

Option 4 ID : 3398938128

Status : Marked For Review

Chosen Option : 1

Q.9 Let $\mathbb{R}$ denote the set of real numbers and X be a non-empty set. Let Y be the set of all functions from $\mathbb{R}$ to X . Then which of the following is true?

1. If X is finite, then Y is countable
2. Y is always infinite
3. If Y is infinite, then Y is uncountable
4. If Y is uncountable, then X is uncountable

Options 1. 1

2. 2

3. 3

Question Type : MCQ

Question ID : 3398932078

Option 1 ID : 3398938081

Option 2 ID : 3398938082

Option 3 ID : 3398938083

Option 4 ID : 3398938084

Status : Not Answered

Chosen Option : --

Q.10 Let $f(t) = t^n + a_{n-1}t^{n-1} + \dots + a_1t + a_0$ be a non-constant even degree polynomial with real coefficients and $a_0 < 0$. Then the polynomial f

1. need not have a real root.
2. has at least two distinct real roots
3. has at least two real roots but need not be distinct
4. can have at most one real root

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932083

Option 1 ID : 3398938101

Option 2 ID : 3398938102

Option 3 ID : 3398938103

Option 4 ID : 3398938104

Status : Marked For Review

Chosen Option : 2

Q.11 Let A and B be 3×3 matrices with real entries.

Let $V_1 = \{v \in \mathbb{R}^3 \mid ABv = 0\}$,

$V_2 = \{v \in \mathbb{R}^3 \mid Bv = 0\}$, and

$V_3 = \{v \in \mathbb{R}^3 \mid Av = 0\}$.

Which of the following is necessarily true?

1. $\dim V_1 = \dim V_2 \Rightarrow A$ is invertible
2. $\dim V_1 = \dim V_3 \Rightarrow A$ is invertible
3. A is invertible $\Rightarrow \dim V_1 = \dim V_2$
4. A is invertible $\Rightarrow \dim V_1 = \dim V_3$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

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Question ID : 3398932085

Option 1 ID : 3398938109

Option 2 ID : 3398938110

Option 3 ID : 3398938111

Option 4 ID : 3398938112

Status : Not Answered

Chosen Option : --

Q.12 Consider the ordered basis $v_1 = (1, 0, 0)$, $v_2 = (1, 1, 0)$, $v_3 = (1, 1, 1)$ of $\mathbb{R}^3$.

What are the co-ordinates of $(1, 2, 3)$ with respect to this basis?

1. $(-1, 1, 3)$
2. $(-1, -1, 3)$
3. $(1, 1, 3)$
4. $(1, -1, 3)$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCO

Question ID : 3398932084

Option 1 ID : 3398938105

Option 2 ID : 3398938106

Option 3 ID : 3398938107

Option 4 ID : 3398938108

Status : Marked For Review

Chosen Option : 2

Q.13 Which of the following sets lies in the region of convergence of

$$\sum_{n=0}^{\infty} (3z - 2i)^{3n}?$$

1. $\left\{x + \frac{2i}{3}; x \in \left(\frac{-1}{3}, \frac{1}{3}\right)\right\}$
2. $\left\{x + \frac{2i}{3}; x \in \left(\frac{-1}{2}, \frac{1}{9}\right)\right\}$
3. $\left\{x + \frac{2i}{3}; x \in \left(\frac{-1}{27}, \frac{20}{27}\right)\right\}$
4. $\left\{x + \frac{2i}{3}; x \in \left(\frac{-5}{6}, \frac{5}{6}\right)\right\}$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCO

Question ID : 3398932092

Option 1 ID : 3398938137

Option 2 ID : 3398938138

Option 3 ID : 3398938139

Option 4 ID : 3398938140

Status : Marked For Review

Chosen Option : 1

Q.14

Let S_5 denote the symmetric group on 5 letters. Which of the following is always true?

1. If $\sigma \in S_5$ is a k -cycle, then σ^i is also a k -cycle for all $2 \leq k \leq 5$ and $i \geq 1$.
2. If $\sigma, \tau \in S_5$ have both order 5, then $\sigma\tau$ also has order 5
3. Any 3-cycle in S_5 can be written as a product of two 5-cycles
4. Any 2-cycle in S_5 can be written as a product of two 5-cycles

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932095

Option 1 ID : 3398938149

Option 2 ID : 3398938150

Option 3 ID : 3398938151

Option 4 ID : 3398938152

Status : Not Answered

Chosen Option : --

Q.15 Let $\alpha \in \mathbb{R}$. Consider $\mathbb{R}$ and $\mathbb{R}^2$ with the usual topology. Consider the set $A = \{(x, y) \in \mathbb{R}^2 / xy = 1\} \cup \{(0, 0)\}$ in $\mathbb{R}^2$ with subspace topology. The maps $f_1: A \rightarrow \mathbb{R}$ and $f_2: A \rightarrow \mathbb{R}$ are given by $f_1(x, y) = x + \alpha$ and $f_2(x, y) = y$. Which of the following statements is true?

1. The map f_2 is continuous but f_1 is not continuous
2. The maps f_1 and f_2 are open maps
3. The maps f_1 and f_2 are closed maps
4. The set $\{(x, y) \in A \mid f_1(x, y) = f_2(x, y)\}$ is closed

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932097

Option 1 ID : 3398938157

Option 2 ID : 3398938158

Option 3 ID : 3398938159

Option 4 ID : 3398938160

Status : Not Answered

Chosen Option : --

Q.16 Let $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ be the open unit disc and $f: \mathbb{D} \rightarrow \mathbb{C}$ a holomorphic function such that $|f(z)| \leq 1$ on $\mathbb{D}$. Suppose that $f(0) = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$. Then

1. $f\left(-\frac{1}{2}\right) = \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$
2. $f\left(-\frac{1}{2}\right) = \frac{1}{\sqrt{3}} - \frac{i}{\sqrt{3}}$
3. $f\left(\frac{1}{2}\right) = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$
4. $f\left(\frac{1}{2}\right) = \frac{1}{\sqrt{3}} + \frac{i}{\sqrt{3}}$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932093

Option 1 ID : 3398938141

Option 2 ID : 3398938142

Option 3 ID : 3398938143

Option 4 ID : 3398938144

Status : Marked For Review

Chosen Option : 3

Q.17 Let f be a non-constant entire function and γ the positively oriented circle $\{z \in \mathbb{C} : |z| = 2\}$. Then

$$\frac{1}{\pi} \int_{\gamma} \frac{f(z)}{z^2 + 1} dz \text{ equals}$$

1. $f(1) + f(-1)$

2. $f(1) - f(-1)$

3. $f(i) + f(-i)$

4. $f(i) - f(-i)$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932090

Option 1 ID : 3398938129

Option 2 ID : 3398938130

Option 3 ID : 3398938131

Option 4 ID : 3398938132

Status : Marked For Review

Chosen Option : 4

Q.18 Let F be a finite field of order q . What is the number of 2-dimensional subspaces of the vector space F^3 over F ?

1. q^3

2. q^2

3. $(q^3 - 1)/(q - 1)$

4. $q^3 - 1$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932096

Option 1 ID : 3398938153

Option 2 ID : 3398938154

Option 3 ID : 3398938155

Q.19 For which value of a among the following does $f(z) = \frac{az+2}{3z+1}$ map the upper half plane $\mathbb{H} = \{z \in \mathbb{C} : z = x + iy, y > 0\}$ to $\mathbb{H}$?

1. 1
2. 3
3. 5
4. 7

Options 1. 1

2. 2
3. 3
4. 4

Question Type : MCO

Question ID : 3398932091

Option 1 ID : 3398938133

Option 2 ID : 3398938134

Option 3 ID : 3398938135

Option 4 ID : 3398938136

Status : Marked For Review

Chosen Option : 4

Q.20 Let R_1 be the ring $\mathbb{Z}/(11\mathbb{Z})$ and let R_2 be the ring $\mathbb{Z}/(13\mathbb{Z})$. Let

A = number of ideals in the product ring $R_1 \times R_2$

B = number of ring homomorphisms $R_1 \rightarrow R_2$ sending 1 to 1

C = number of ring homomorphisms $R_2 \rightarrow R_1$ sending 1 to 1

What is $A + B + C$?

1. 3
2. 4
3. 6
4. 8

Options 1. 1

2. 2
3. 3
4. 4

Question Type : MCO

Question ID : 3398932094

Option 1 ID : 3398938145

Option 2 ID : 3398938146

Option 3 ID : 3398938147

Option 4 ID : 3398938148

Status : Not Answered

Chosen Option : --

Q.21

Let $|f(x)| \leq 4|x|$ for $x \in \mathbb{R}$. The largest possible value of $|x'(t)|$ where $x'(t) = f(x(t))$, $t > 0$, $x(0) = 3$, is

1. $3e^4$
2. $4e^3$
3. $12e^3$
4. $12e^4$

Options

1. 1
2. 2
3. 3
4. 4

Question Type : MCO

Question ID : 3398932098

Option 1 ID : 3398938161

Option 2 ID : 3398938162

Option 3 ID : 3398938163

Option 4 ID : 3398938164

Status : Not Answered

Chosen Option : --

Q.22 Assume that $a, b \in \mathbb{R} \setminus \{0\}$ and $a^2 \neq b^2$. Suppose that the Gauss-Seidel method is used to solve the system of equations

$$\begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Then the set of all values of (a, b) such that the method converges for every choice of initial vector is

1. $\{(a, b) \mid a^2 < b^2\}$
2. $\{(a, b) \mid a < |b|\}$
3. $\{(a, b) \mid |b| < |a|\}$
4. $\{(a, b) \mid a^2 + b^2 < 1\}$

Options

1. 1
2. 2
3. 3
4. 4

Question Type : MCO

Question ID : 3398932102

Option 1 ID : 3398938177

Option 2 ID : 3398938178

Option 3 ID : 3398938179

Option 4 ID : 3398938180

Status : Not Answered

Chosen Option : --

Q.23

If $\varphi: [0, 1] \rightarrow \mathbb{R}$ is continuous and satisfies

$$\int_0^x (x-t)^2 \varphi(t) dt = x^4 + x^5,$$

then $\varphi(1) =$

1. 18
2. 30
3. 42
4. 48

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCO

Question ID : 3398932104

Option 1 ID : 3398938185

Option 2 ID : 3398938186

Option 3 ID : 3398938187

Option 4 ID : 3398938188

Status : Marked For Review

Chosen Option : 3

Q.24 Let $u(x, y, z) = C_1$ and $v(x, y, z) = C_2$ be two independent families of smooth surfaces. Let $P, Q, R \in C^1(\mathbb{R}^3)$ and for any $\xi \in \mathbb{R}^3$, that lies on the curve of intersection of $u(x, y, z) = C_1$ and $v(x, y, z) = C_2$, $(P(\xi), Q(\xi), R(\xi))$ is in the direction of the tangent to the curve of intersection at ξ . Then a general solution z of $P \frac{\partial z}{\partial x} + Q \frac{\partial z}{\partial y} = R$ is implicitly given by

1. $u + v = f(u + v)$ for every $f \in C^1(\mathbb{R})$
2. $uv = f(u)$ for every $f \in C^1(\mathbb{R})$
3. $u = f(uv)$ for every $f \in C^1(\mathbb{R})$
4. $uv = f(uv)$ for every $f \in C^1(\mathbb{R})$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCO

Question ID : 3398932101

Option 1 ID : 3398938173

Option 2 ID : 3398938174

Option 3 ID : 3398938175

Option 4 ID : 3398938176

Status : Not Answered

Chosen Option : --

Q.25

Let $\bar{q} = (q_1, \dots, q_n)$ be the generalized coordinates and $\bar{p} = (p_1, \dots, p_n)$ be the generalized momenta of a system. Let the Poisson bracket of two quantities f, g be denoted by $[f, g]_{\bar{q}, \bar{p}}$ and the Hamiltonian of the system be H . Then which of the following statements is true?

1. If $\frac{\partial f}{\partial t} = 0, [H, f]_{\bar{q}, \bar{p}} = 0$ then f is a conserved quantity
2. If f is a conserved quantity then $[f, p_i]_{\bar{q}, \bar{p}} = 0, 1 \leq i \leq n$
3. If f is a conserved quantity then $[f, q_i]_{\bar{q}, \bar{p}} = 0, 1 \leq i \leq n$
4. There exists a canonical transformation $(\bar{q}, \bar{p}) \rightarrow (\bar{Q}, \bar{P})$ such that $[f, g]_{\bar{q}, \bar{p}} \neq [f, g]_{\bar{Q}, \bar{P}}$ for some f, g

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932105

Option 1 ID : 3398938189

Option 2 ID : 3398938190

Option 3 ID : 3398938191

Option 4 ID : 3398938192

Status : Not Answered

Chosen Option : --

Q.26 The 2nd order partial differential equation

$$e^x \frac{\partial^2 u}{\partial x^2} + (x^2 + y^2 - 1) \frac{\partial^2 u}{\partial y^2} = 0 \text{ is}$$

1. elliptic inside the unit circle with centre at (0,0)
2. hyperbolic outside the unit circle with centre at (0,0)
3. elliptic outside the unit circle with centre at (0,0)
4. parabolic for $x, y \in \mathbb{R}$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932100

Option 1 ID : 3398938169

Option 2 ID : 3398938170

Option 3 ID : 3398938171

Option 4 ID : 3398938172

Status : Marked For Review

Chosen Option : 3

Q.27

The critical point (0,0) for the system of equations

$$x'(t) = x^2 + y^2 - 2x$$

$$y'(t) = 3x^2 - x + 3y$$

is a

1. stable point
2. source point
3. saddle point
4. spiral stable point

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932099

Option 1 ID : 3398938165

Option 2 ID : 3398938166

Option 3 ID : 3398938167

Option 4 ID : 3398938168

Status : Not Answered

Chosen Option : --

Q.28 The extremal for the following functional

$$\int_0^1 (\alpha t x(t) + (x'(t))^2) dt, \quad \alpha \neq 0,$$

where $x(0) = 1, x'(0) = 0$, is

1. $\frac{\alpha}{12} t^3 + 1$
2. $t^3 + \alpha t^2 + 1$
3. $t^2 + t + 1$
4. $t^4 + \alpha t^2 + 1$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932103

Option 1 ID : 3398938181

Option 2 ID : 3398938182

Option 3 ID : 3398938183

Option 4 ID : 3398938184

Status : Marked For Review

Chosen Option : 1

Q.29 Let X_1, X_2 be i.i.d. random variables from an exponential distribution with mean $\frac{1}{\theta}$ where $\theta > 0$. Suppose that the prior distribution for θ is exponential with mean 1. Then the Bayes estimator for θ with respect to the squared error loss function is

1. $\frac{X_1 + X_2}{2} + 1$
2. $\frac{2}{X_1 + X_2}$
3. $\frac{3}{X_1 + X_2 + 1}$
4. $\frac{1}{3(X_1 + X_2)}$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932112

Option 1 ID : 3398938217

Option 2 ID : 3398938218

Option 3 ID : 3398938219

Option 4 ID : 3398938220

Status : Not Answered

Chosen Option : --

Q.30 Let $N, X_1, X_2, X_3, \dots$ be independent random variables where N has Poisson (λ) distribution and X_k has normal distribution with mean k and variance k^2 for each $k \geq 1$. Then, the variance of X_{N+1} is

1. λ
2. $\lambda^2 + 3\lambda + 1$
3. $\lambda^2 + 4\lambda + 2$
4. $\lambda^2 + 4\lambda + 1$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932107

Option 1 ID : 3398938197

Option 2 ID : 3398938198

Option 3 ID : 3398938199

Option 4 ID : 3398938200

Status : Not Answered

Chosen Option : --

Q.31

Consider the set Ω of all 4-tuples (x_1, x_2, x_3, x_4) of integers such that each $x_i \geq 0$ and $x_1 + x_2 + x_3 + x_4 = 90$. If a point is selected uniformly at random from Ω , the conditional probability that $x_1 \geq 1$ given that $x_3 \geq 44$ and $x_4 \geq 44$ equals

1. $\frac{1}{4}$
2. $\frac{1}{3}$
3. $\frac{2}{5}$
4. $\frac{1}{8}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCO

Question ID : 3398932106

Option 1 ID : 3398938193

Option 2 ID : 3398938194

Option 3 ID : 3398938195

Option 4 ID : 3398938196

Status : Not Answered

Chosen Option : --

Q.32 In a block design with b blocks and v treatments, every treatment pair occurs exactly once in each block. Then the resulting design will be

1. connected and incomplete
2. not connected and complete
3. connected and complete
4. not connected and incomplete

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MCO

Question ID : 3398932116

Option 1 ID : 3398938233

Option 2 ID : 3398938234

Option 3 ID : 3398938235

Option 4 ID : 3398938236

Status : Not Answered

Chosen Option : --

Q.33 In a simple linear regression model, assume that the random errors $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n$ are uncorrelated and homoscedastic. Then we may conclude that the residuals $e_1, e_2, \dots, e_n$ are

1. uncorrelated and homoscedastic
2. correlated and homoscedastic
3. uncorrelated and heteroscedastic
4. correlated and heteroscedastic

Options 1. 1
2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932113

Option 1 ID : 3398938221

Option 2 ID : 3398938222

Option 3 ID : 3398938223

Option 4 ID : 3398938224

Status : Not Answered

Chosen Option : --

Q.34 Let $X_1, X_2, \dots, X_n$ be a random sample from $N(\theta, 1)$ distribution. For testing $H_0: \theta = 0$ against $H_1: \theta < 0$, if the UMP test is used and if the observed sample mean is 1, then

1. the test is biased
2. H_0 is rejected at 5% level of significance
3. H_0 is NOT rejected at 1% level of significance
4. p -value of the test is less than $\frac{1}{2}$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932111

Option 1 ID : 3398938213

Option 2 ID : 3398938214

Option 3 ID : 3398938215

Option 4 ID : 3398938216

Status : Not Answered

Chosen Option : --

Q.35 Consider a Markov chain with state space $S = \{0, 1, \dots, 1000\}$ and transition probabilities given by $p_{i,i+1} = 1$ for $0 \leq i \leq 999$ and $p_{1000,1000} = p_{1000,0} = \frac{1}{2}$.

Then

1. $\lim_{n \rightarrow \infty} p_{ij}^{(n)} = \frac{1}{1000}$ for all i, j
2. $\lim_{n \rightarrow \infty} p_{ij}^{(n)} = \frac{1}{1000}$ for all $i, j \leq 999$
3. $\lim_{n \rightarrow \infty} p_{ij}^{(n)} = \frac{1}{1001}$ for all i, j
4. $\lim_{n \rightarrow \infty} p_{ij}^{(n)} = \frac{1}{1002}$ for all $i, j \leq 999$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932108

Option 1 ID : 3398938201

Option 2 ID : 3398938202

Option 3 ID : 3398938203

Option 4 ID : 3398938204

Q.36 The goal is to estimate the unknown parameter μ as accurately as possible.

You are offered a choice between two data sets. Assume $n \geq 2$.

(A): $X_1, X_2, \dots, X_n$ i.i.d. normal with mean μ and variance 1

(B): $Y_1, Y_2, \dots, Y_n$ i.i.d. normal with mean 2μ and variance 2

Then

1. data (A) is preferable
2. data (B) is preferable
3. both data sets are equally good
4. which data set is preferable depends on the value of n

Options

1. 1
2. 2
3. 3
4. 4

Question Type : MCO

Question ID : 3398932117

Option 1 ID : 3398938237

Option 2 ID : 3398938238

Option 3 ID : 3398938239

Option 4 ID : 3398938240

Status : Not Answered

Chosen Option : --

Q.37 Let $X_1, X_2, \dots, X_n$ be i.i.d. Uniform($0, \theta$) random variables where $\theta > 0$. Then, a consistent estimator of θ is

1. $\min(X_1, \dots, X_n) + \frac{1}{n}$
2. $\frac{1}{n} (X_1 + \dots + X_n)$
3. $\max(X_1, \dots, X_n) - \frac{1}{n}$
4. $\frac{n}{X_1 + \dots + X_n}$

Options

1. 1
2. 2
3. 3
4. 4

Question Type : MCO

Question ID : 3398932110

Option 1 ID : 3398938209

Option 2 ID : 3398938210

Option 3 ID : 3398938211

Option 4 ID : 3398938212

Status : Not Answered

Chosen Option : --

Q.38

X is a Poisson random variable with parameter 20. The conditional distribution of Y given $X = k$ is Binomial $\left(k, \frac{1}{2}\right)$ for $k \geq 1$ and $P(Y = 0|X = 0) = 1$. Then the distribution (unconditional) of Y is

1. Poisson(10)
2. Poisson(20)
3. Poisson(40)
4. Binomial $\left(20, \frac{1}{2}\right)$

Options 1. 1

2. 2
3. 3
4. 4

Question Type : MCO

Question ID : 3398932109

Option 1 ID : 3398938205

Option 2 ID : 3398938206

Option 3 ID : 3398938207

Option 4 ID : 3398938208

Status : Not Answered

Chosen Option : --

Q.39 From a population of $N = nk$ units, a sample of size n is drawn using linear systematic sampling scheme. Then the inclusion probability of the i^{th} unit for $i = 1, \dots, N$ is

1. $\frac{1}{N}$
2. $\frac{1}{n}$
3. $\frac{1}{k}$
4. $\frac{k!}{N!}$

Options 1. 1

2. 2
3. 3
4. 4

Question Type : MCO

Question ID : 3398932115

Option 1 ID : 3398938229

Option 2 ID : 3398938230

Option 3 ID : 3398938231

Option 4 ID : 3398938232

Status : Not Answered

Chosen Option : --

Q.40

Suppose $B = ((b_{ij})) \sim W_p(k, \Sigma)$ (Wishart distribution), where $p = 7$, $k = 12$

and $\Sigma = ((\sigma_{ij}))$ is a positive definite matrix. Then the distribution of

$$\frac{\sum_{i=1}^p \sum_{j=1}^p b_{ij}}{\sum_{i=1}^p \sum_{j=1}^p \sigma_{ij}} \text{ is}$$

1. χ_1^2
2. χ_7^2
3. χ_{12}^2
4. χ_5^2

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MCQ

Question ID : 3398932114

Option 1 ID : 3398938225

Option 2 ID : 3398938226

Option 3 ID : 3398938227

Option 4 ID : 3398938228

Status : Not Answered

Chosen Option : --

Section : Part C Mathematical Sciences

Q.1 Let A be a 3×3 matrix over $\mathbb{C}$. Let $\omega = e^{2\pi i/3}$. Which of the following are true?

1. If $A^3 = I$, then the eigenvalues of A are $1, \omega, \omega^2$
2. If $A^3 = I$, then every eigenvalue of A is in the set $\{1, \omega, \omega^2\}$
3. If the eigenvalues of A are $1, \omega, \omega^2$, then $A^3 = I$
4. If every eigenvalue of A is in the set $\{1, \omega, \omega^2\}$, then $A^3 = I$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932132

Option 1 ID : 3398938297

Option 2 ID : 3398938298

Option 3 ID : 3398938299

Option 4 ID : 3398938300

Status : Marked For Review

Chosen Option : 2

Q.2

Consider the infinite series

$$\sum_{n=1}^{\infty} (-1)^n \frac{\cos(nx)}{n^2 + x^2}, \quad \text{where } x \in \mathbb{R}.$$

The largest set on which it converges uniformly is:

1. $\mathbb{R}$
2. $\mathbb{R} \setminus \{0\}$
3. $\mathbb{R} \setminus \{n\pi : n \in \mathbb{Z}\}$
4. $[0, 2\pi]$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932120

Option 1 ID : 3398938249

Option 2 ID : 3398938250

Option 3 ID : 3398938251

Option 4 ID : 3398938252

Status : Marked For Review

Chosen Option : 1

Q.3 Let V be an n dimensional inner product space over $\mathbb{R}$, where $n \geq 2$. A linear map $T: V \rightarrow V$ is said to be an orthogonal map if $\langle Tv, w \rangle = \langle v, T^{-1}w \rangle$ for all $v, w \in V$. Which of the following are true?

1. T is orthogonal $\Rightarrow T$ is diagonalizable over $\mathbb{R}$
2. T diagonalizable over $\mathbb{R} \Rightarrow T$ is orthogonal
3. T is orthogonal $\Leftrightarrow \langle Tv, Tw \rangle = \langle v, w \rangle$ for all $v, w \in V$
4. T is orthogonal $\Leftrightarrow \langle Tv, Tv \rangle = \langle v, v \rangle$ for all $v \in V$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932134

Option 1 ID : 3398938305

Option 2 ID : 3398938306

Option 3 ID : 3398938307

Option 4 ID : 3398938308

Status : Not Answered

Chosen Option : --

Q.4

Let I be any interval in $\mathbb{R}$. Let $f, g: I \rightarrow \mathbb{R}$ be uniformly continuous functions. If $h(x) = f(x)g(x)$, then h is uniformly continuous if

1. either f or g is bounded
2. I is compact
3. I is bounded
4. I is closed

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSO

Question ID : 3398932121

Option 1 ID : 3398938253

Option 2 ID : 3398938254

Option 3 ID : 3398938255

Option 4 ID : 3398938256

Status : Not Answered

Chosen Option : --

Q.5 Let (x_n) be a sequence of real numbers with $|x_n| > 2$. Then which of the following are true?

1. $\lim_{n \rightarrow \infty} x_n = \infty \Rightarrow \lim_{n \rightarrow \infty} \left(1 + \frac{1}{x_n}\right)^{x_n} = \infty$
2. $\lim_{n \rightarrow \infty} x_n = \infty \Rightarrow \lim_{n \rightarrow \infty} \left(1 + \frac{1}{x_n}\right)^{x_n} = 1$
3. $\lim_{n \rightarrow \infty} x_n = -\infty \Rightarrow \lim_{n \rightarrow \infty} \left(1 + \frac{1}{x_n}\right)^{x_n} = 0$
4. $\lim_{n \rightarrow \infty} x_n = -\infty \Rightarrow \lim_{n \rightarrow \infty} \left(1 + \frac{1}{x_n}\right)^{x_n} > 1$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSO

Question ID : 3398932118

Option 1 ID : 3398938241

Option 2 ID : 3398938242

Option 3 ID : 3398938243

Option 4 ID : 3398938244

Status : Marked For Review

Chosen Option : 4

Q.6 For $x, y \in \mathbb{R}^2$ define $\langle x, y \rangle = x^t A y$, where A is a 2×2 matrix with real entries. For which of the following choices of A does this define an inner product on $\mathbb{R}^2$?

1. $A = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$
2. $A = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$
3. $A = \begin{bmatrix} 2 & 2 \\ 2 & 1 \end{bmatrix}$
4. $A = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932133

Option 1 ID : 3398938301

Option 2 ID : 3398938302

Option 3 ID : 3398938303

Option 4 ID : 3398938304

Status : Not Answered

Chosen Option : --

Q.7 Let V be a finite dimensional vector space over a field K . Then, which of the following are true?

1. V is not a union of two proper subspaces
2. If V has exactly two subspaces, then V is isomorphic to K as a vector space
3. V is not a union of finitely many proper subspaces
4. If $\dim V > 1$, then the intersection of all non-zero subspaces of V is a non-zero subspace

Options

1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932129

Option 1 ID : 3398938285

Option 2 ID : 3398938286

Option 3 ID : 3398938287

Option 4 ID : 3398938288

Status : Not Answered

Chosen Option : --

Q.8 Let v_1, v_2, w_1, w_2 be vectors in $\mathbb{R}^4$ such that $v_1, v_2, v_1 + w_1, v_2 + w_2$ is a basis of $\mathbb{R}^4$. Which of the following are bases of $\mathbb{R}^4$?

1. $v_1, v_2, 2v_1 + 3w_2, 5v_2 - 3w_1$
2. $v_1, v_2, 2v_1 + 2w_1, v_1 + w_2$
3. $v_1 + v_2, v_2, w_1, 2v_1 + 5v_2 - 4w_1$
4. $v_1 + v_2, v_2, w_1 + w_2, 2v_1 + v_2 + w_1 + w_2$

Options

1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932130

Option 1 ID : 3398938289

Option 2 ID : 3398938290

Option 3 ID : 3398938291

Option 4 ID : 3398938292

Status : Not Answered

Chosen Option : -- 27th Dec 2019 - {31}

Q.9 Let (X, d) be a metric space. Which of the following are NOT in general a metric on X ?

1. $\rho(x, y) = (d(x, y))^2$
2. $\rho(x, y) = \sqrt{d(x, y)}$
3. $\rho(x, y) = \min\{d(x, y), 1\}$
4. $\rho(x, y) = \begin{cases} 0 & \text{if } d(x, y) = 0 \\ 1 & \text{if } d(x, y) > 0 \end{cases}$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932127

Option 1 ID : 3398938277

Option 2 ID : 3398938278

Option 3 ID : 3398938279

Option 4 ID : 3398938280

Status : Marked For Review

Chosen Option : 2,3,4

Q.10 Which of the following matrices are diagonalizable over $\mathbb{C}$?

1. $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$

3. $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4. $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932128

Option 1 ID : 3398938281

Option 2 ID : 3398938282

Option 3 ID : 3398938283

Option 4 ID : 3398938284

Status : Marked For Review

Chosen Option : 2,4

Q.11

Let T be a 3×3 matrix with entries in $\mathbb{R}$. Consider the system of linear

equations $T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$. Which of the following are true?

1. Rank of $T \leq 2 \Rightarrow$ the system has a non-zero solution
2. Rank of $T \leq 1 \Rightarrow$ the system has infinitely many solutions
3. Rank of $T \geq 2 \Rightarrow$ any two solutions of the system are linearly dependent
4. Rank of $T \geq 1 \Rightarrow$ every $\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} \in \mathbb{R}^3$ is a solution of the system

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932135

Option 1 ID : 3398938309

Option 2 ID : 3398938310

Option 3 ID : 3398938311

Option 4 ID : 3398938312

Status : Marked For Review

Chosen Option : 4

Q.12 Consider $f_n(x) = \frac{1}{n}e^{-x/n}$ for $x \in [0, \infty)$. Then which of the following are true?

1. $\{f_n\}$ converges uniformly
2. The sequence of derivatives $\{f_n'\}$ converges uniformly
3. $\lim_{n \rightarrow \infty} \int_0^{\infty} f_n(x) dx \neq \int_0^{\infty} \left(\lim_{n \rightarrow \infty} f_n(x) \right) dx$
4. $\lim_{n \rightarrow \infty} \int_0^{\infty} f_n(x) dx = \int_0^{\infty} \left(\lim_{n \rightarrow \infty} f_n(x) \right) dx$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932123

Option 1 ID : 3398938261

Option 2 ID : 3398938262

Option 3 ID : 3398938263

Option 4 ID : 3398938264

Status : Marked For Review

Chosen Option : 1,2,3

Q.13

Let T_1, T_2 be linear operators on $\mathbb{R}^2$ such that $T_1 T_2 = 0$. Which of the following are necessarily true?

1. 0 is an eigenvalue of T_1 or 0 is an eigenvalue of T_2
2. There exists a nonzero vector $w \in \mathbb{R}^2$ such that $T_1 w = T_2 w = 0$
3. $T_2 T_1 = 0$
4. $T_1 = 0$ or $T_2 = 0$

Options

1. 1
2. 2
3. 3
4. 4

Question Type : MSQ

Question ID : 3398932131

Option 1 ID : 3398938293

Option 2 ID : 3398938294

Option 3 ID : 3398938295

Option 4 ID : 3398938296

Status : Not Answered

Chosen Option : --

Q.14 Let $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ be defined as $f(x, y) = x|x| + |y|$. Then which of the following are true?

1. f is differentiable at $(0, 0)$
2. f is not differentiable at $(0, 0)$ but all its directional derivatives at $(0, 0)$ exist
3. $\frac{\partial f}{\partial x}(0, 0)$ exists and equals 0
4. $\frac{\partial f}{\partial y}(0, 0)$ exists and equals 0

Options

1. 1
2. 2
3. 3
4. 4

Question Type : MSQ

Question ID : 3398932126

Option 1 ID : 3398938273

Option 2 ID : 3398938274

Option 3 ID : 3398938275

Option 4 ID : 3398938276

Status : Not Answered

Chosen Option : --

Q.15 Let $f: [a, b) \rightarrow \mathbb{R}$ be a continuous function. Define $F: [a, b) \rightarrow \mathbb{R}$ by

$$F(x) = \max_{t \in [a, x]} f(t)$$

Then which of the following are true?

1. F need not be continuous
2. F is necessarily monotonic
3. F is necessarily bounded
4. F is Riemann integrable on $[a, x]$ for every $x \in [a, b)$

Options

1. 1
2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932122

Option 1 ID : 3398938257

Option 2 ID : 3398938258

Option 3 ID : 3398938259

Option 4 ID : 3398938260

Status : Not Answered

Chosen Option : --

Q.16 Let $P(x)$ be a polynomial with real coefficients and of degree $n \geq 1$. Then which of the following are true?

1. If n is odd, then $P(x)$ has a real root
2. If $P(x)$ has only real roots then its derivative $P'(x)$ has only real roots
3. If $P'(x)$ has only real roots, then $P(x)$ has only real roots
4. If n is odd, then $P(x)$ takes every real value as x varies over reals

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932119

Option 1 ID : 3398938245

Option 2 ID : 3398938246

Option 3 ID : 3398938247

Option 4 ID : 3398938248

Status : Marked For Review

Chosen Option : 1

Q.17 Let $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ be defined as

$$f(x, y) = \begin{cases} \frac{xy^3}{x^2 + y^6} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

Then which of the following are true?

1. f is bounded on $\mathbb{R}^2$
2. the restriction of f to each line of the form $y = mx$ is continuous at $(0, 0)$
3. f is continuous at $(0, 0)$
4. f is not continuous at $(0, 0)$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932125

Option 1 ID : 3398938269

Option 2 ID : 3398938270

Option 3 ID : 3398938271

Option 4 ID : 3398938272

Status : Marked For Review

Q.18 For any interval I of $\mathbb{R}$ let $BV(I)$ denote the space of all functions on I which are of bounded variation. Then which of the following statements are true?

1. If f, g belong to $BV([a, b])$ then the product $f \cdot g$ belongs to $BV([a, b])$
2. If $\{f_n\}$ is a sequence in $BV([a, b])$ that converges to f pointwise, then f belongs to $BV([a, b])$
3. If $f: [0, \infty) \rightarrow \mathbb{R}$ is a function such that f restricted to $[n, n+1]$ belongs to $BV([n, n+1])$ for $n = 0, 1, 2, \dots$, then f belongs to $BV([0, \infty))$
4. The function $f(x) = \begin{cases} \frac{e^x - 1}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$ belongs to $BV([a, b])$ for any $a < b$.

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932124

Option 1 ID : 3398938265

Option 2 ID : 3398938266

Option 3 ID : 3398938267

Option 4 ID : 3398938268

Status : Marked For Review

Chosen Option : 1,2,4

Q.19 Let $K \subset \mathbb{R}^2$ be an arbitrary non-empty set and let $\pi_1(x, y) = x$ and $\pi_2(x, y) = y$ be the projections from $\mathbb{R}^2$ onto the first and second coordinates respectively. Which of the following statements are true?

1. If K is compact, then $\pi_1(K)$ and $\pi_2(K)$ are both compact
2. If $\pi_1(K)$ and $\pi_2(K)$ are both compact, then K is compact
3. If K is connected, then both $\pi_1(K)$ and $\pi_2(K)$ are connected
4. If $\pi_1(K)$ and $\pi_2(K)$ are both connected, then K is connected

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932147

Option 1 ID : 3398938357

Option 2 ID : 3398938358

Option 3 ID : 3398938359

Option 4 ID : 3398938360

Status : Not Answered

Chosen Option : --

Q.20

Let X be a topological space and let A and B be non-empty subsets of X . Suppose $A \cup B$ and $A \cap B$ are connected. Then, which of the following statements are true?

1. At least one of A and B is connected
2. If A is closed and B is open, then both A and B are connected
3. If A and B are open sets, then both A and B are connected
4. If A and B are closed sets, then both A and B are connected

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ

Question ID : 3398932146

Option 1 ID : 3398938353

Option 2 ID : 3398938354

Option 3 ID : 3398938355

Option 4 ID : 3398938356

Status : Not Answered

Chosen Option : --

Q.21 How many 3-sylow subgroups does the symmetric group S_4 have?

1. 8
2. 4
3. 2
4. 1

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ

Question ID : 3398932143

Option 1 ID : 3398938341

Option 2 ID : 3398938342

Option 3 ID : 3398938343

Option 4 ID : 3398938344

Status : Marked For Review

Chosen Option : 2

Q.22 Let f be a holomorphic function on the open unit disc $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$

such that $f\left(\frac{1}{n}\right) = \frac{2n+1}{2n-1}$ for $n = 1, 2, \dots$. Let $\Im(w)$ denote the imaginary part of a complex number w . Then, which of the following are true?

1. $\Im(f(i/2)) > 0$ and $\Im(f(-i/2)) < 0$
2. $\Im(f(i/2)) < 0$ and $\Im(f(-i/2)) > 0$
3. $\Im(f(i/2)) > 0$ and $\Im(f(-i/2)) > 0$
4. $\Im(f(i/2)) < 0$ and $\Im(f(-i/2)) < 0$

Options 1. 1
2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932137

Option 1 ID : 3398938317

Option 2 ID : 3398938318

Option 3 ID : 3398938319

Option 4 ID : 3398938320

Status : Not Answered

Chosen Option : --

Q.23 Which of the following statements are correct?

1. The fields $\mathbb{Q}(\sqrt{5})$ and $\mathbb{Q}(\sqrt{7})$ are isomorphic as vector spaces over $\mathbb{Q}$
2. The fields $\mathbb{Q}(\sqrt{5})$ and $\mathbb{Q}(\sqrt{7})$ are isomorphic as fields
3. The Galois group of $\mathbb{Q}(\sqrt{5})/\mathbb{Q}$ is isomorphic to the Galois group of $\mathbb{Q}(\sqrt{7})/\mathbb{Q}$
4. The fields $\mathbb{Q}(\sqrt{5}, \sqrt{7})$ and $\mathbb{Q}(\sqrt{5} + \sqrt{7})$ are isomorphic as fields

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932145

Option 1 ID : 3398938349

Option 2 ID : 3398938350

Option 3 ID : 3398938351

Option 4 ID : 3398938352

Status : Not Answered

Chosen Option : --

Q.24 Let γ be the positively oriented unit circle in the complex plane. The line integral

$$\oint_{\gamma} e^{2z} dz$$

equals

1. 0
2. 1
3. $4\pi i$
4. $-2\pi i$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932136

Option 1 ID : 3398938313

Option 2 ID : 3398938314

Option 3 ID : 3398938315

Q.25 Let $\mathbb{D}^* = \{z \in \mathbb{C} : 0 < |z| < 1\}$ be the punctured unit disc in the complex plane.

Suppose $f: \mathbb{D}^* \rightarrow \mathbb{C}$ is a holomorphic function that satisfies $|f(z)| \leq \frac{1}{|z|}$ for $z \in \mathbb{D}^*$. Such an f

1. necessarily extends to a holomorphic function on the unit disc
2. necessarily has an essential singularity at $z = 0$
3. has at most a simple pole at $z = 0$
4. can have a pole of order 2 at $z = 0$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSO

Question ID : 3398932138

Option 1 ID : 3398938321

Option 2 ID : 3398938322

Option 3 ID : 3398938323

Option 4 ID : 3398938324

Status : Not Answered

Chosen Option : --

Q.26 The number of $(x_1, x_2, x_3) \in \mathbb{Z}^3$ such that $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$ and such that $x_1 + x_2 + x_3 = 9$ is

1. 55
2. odd
3. $\binom{9}{3}$
4. divisible by 3

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSO

Question ID : 3398932141

Option 1 ID : 3398938333

Option 2 ID : 3398938334

Option 3 ID : 3398938335

Option 4 ID : 3398938336

Status : Answered

Chosen Option : 2,4

Q.27

Let $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ be the open unit disc in $\mathbb{C}$ and let $f: \mathbb{D} \rightarrow \mathbb{D}$ be a non-constant holomorphic function. Suppose that $f(1/3) = 0$. Which of the following are possible values of $\left|f\left(\frac{1}{2}\right)\right|$?

1. $\frac{4}{5}$
2. $\frac{3}{5}$
3. $\frac{2}{5}$
4. $\frac{1}{5}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ

Question ID : 3398932139

Option 1 ID : 3398938325

Option 2 ID : 3398938326

Option 3 ID : 3398938327

Option 4 ID : 3398938328

Status : Not Answered

Chosen Option : --

Q.28 Let $I = \langle xy - 1 \rangle \subseteq \mathbb{C}[x, y]$ be an ideal. Which of the following are true?

1. I is a maximal ideal of $\mathbb{C}[x, y]$
2. I is a prime ideal of $\mathbb{C}[x, y]$
3. $\mathbb{C}[x, y]/I$ has infinitely many prime ideals
4. $\mathbb{C}[x, y]/I$ is a finite set

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ

Question ID : 3398932144

Option 1 ID : 3398938345

Option 2 ID : 3398938346

Option 3 ID : 3398938347

Option 4 ID : 3398938348

Status : Not Answered

Chosen Option : --

Q.29 Let $\mathbb{R}^*$ be the multiplicative group of nonzero real numbers. Which of the following statements are true?

1. If $H \subseteq \mathbb{R}^*$ is a subgroup such that $x^2 \in H$ for all $x \in \mathbb{R}^*$, then $H = \mathbb{R}^*$
2. If $H \subseteq \mathbb{R}^*$ is a subgroup such that $x^3 \in H$ for all $x \in \mathbb{R}^*$, then $H = \mathbb{R}^*$
3. There is no subgroup of $\mathbb{R}^*$ of index 2
4. There is no subgroup of $\mathbb{R}^*$ of index 3

Options 1. 1
2. 2
3. 3

Question Type : MSQ

Question ID : 3398932140

Option 1 ID : 3398938329

Option 2 ID : 3398938330

Option 3 ID : 3398938331

Option 4 ID : 3398938332

Status : Not Answered

Chosen Option : --

Q.30 Let p, q be prime numbers such that $q - 1$ is divisible by p . How many distinct group homomorphisms $f: \mathbb{Z}/p\mathbb{Z} \rightarrow (\mathbb{Z}/q\mathbb{Z})^*$ are there? Here $(\mathbb{Z}/q\mathbb{Z})^*$ denotes the group of units in the ring $\mathbb{Z}/q\mathbb{Z}$.

1. 0
2. 1
3. $\frac{q-1}{p}$
4. p

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932142

Option 1 ID : 3398938337

Option 2 ID : 3398938338

Option 3 ID : 3398938339

Option 4 ID : 3398938340

Status : Marked For Review

Chosen Option : 4

Q.31 Consider a circular disc in the xy -plane with the center at $(0,0)$ and radius one. A simple pendulum of length ℓ and mass m is attached to the rim of the disc. Suppose the disc is rotating with a constant angular velocity ω about the origin and the pendulum is moving in the same plane. Assume that the gravitational force is acting along the positive y -axis and $\theta(t)$ denotes the angle of deviation of the pendulum from the mean position at time t . Then a Lagrangian of the system is

1. $L = \frac{1}{2} ml^2 \dot{\theta}^2 + ml\omega^2 \sin(\theta t - \omega t) + mgl \cos \theta$
2. $L = \frac{1}{2} ml^2 \dot{\theta}^2 + ml\omega \dot{\theta} \sin(\theta - \omega t) + mgl \cos \theta$
3. $L = \frac{1}{2} m(l^2 \dot{\theta}^2 + \omega^2) + ml\omega \dot{\theta} \sin(\theta - \omega t) + mgl \cos \theta - mg \sin \omega t$
4. $L = \frac{1}{2} ml^2 \dot{\theta}^2 + ml\omega \dot{\theta}^2 \sin(\theta t - \omega t) + mgl \sin \theta$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932159

Option 1 ID : 3398938405

Q.32 Consider the problem of minimizing /maximizing

$$J[y] = \int_0^{\pi/8} (y'^2 + 2yy' - 16y^2)dx,$$

subject to $y(0) = 0$, $y\left(\frac{\pi}{8}\right) = 1$. Then

1. every extremizer of J is a minimizer
2. every extremizer of J is a maximizer
3. there exists an extremizer of J which is not a minimizer
4. there exists an extremizer of J which is not a maximizer

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932156

Option 1 ID : 3398938393

Option 2 ID : 3398938394

Option 3 ID : 3398938395

Option 4 ID : 3398938396

Status : Marked For Review

Chosen Option : 3,4

Q.33 Consider the integral equation

$$\varphi(x) - \lambda \int_0^{2\pi} \cos(x-t)\varphi(t)dt = f(x), x \in [0, 2\pi],$$

where $f \in C[0, 2\pi]$.

Then which of the following statements are true?

1. If $\lambda = 10$ and $f(x) = \cos x$, then the integral equation has a solution
2. If $\lambda = \frac{1}{\pi}$ and $f(x) = \cos x$, then the integral equation has no solution
3. For every $\lambda \in \mathbb{R}$ and $f(x) = \cos 2x$, the integral equation has a solution
4. For every $\lambda \in \mathbb{R}$ and $f(x) = \sin 2x$, the integral equation has unique solution

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932158

Option 1 ID : 3398938401

Option 2 ID : 3398938402

Option 3 ID : 3398938403

Option 4 ID : 3398938404

Status : Not Answered

Chosen Option : --

Q.34 Consider the first order initial value problem $y'(x) = -y(x), x > 0, y(0) = 1$ and the corresponding numerical scheme $4\left(\frac{y_{n+1}-y_{n-1}}{2h}\right) - 3\left(\frac{y_{n+1}-y_n}{h}\right) = -y_n$, with $y_0 = 1, y_1 = e^{-h}$, where h is the step size. Then which of the following statements are true?

1. The order of the scheme is 1
2. The order of the scheme is 2
3. $|y_n - y(nh)| \rightarrow \infty$ as $n \rightarrow \infty$
4. $|y_n - y(nh)| \rightarrow 0$ as $n \rightarrow \infty$

Options

1. 1
2. 2
3. 3
4. 4

Question Type : MSQ

Question ID : 3398932154

Option 1 ID : 3398938385

Option 2 ID : 3398938386

Option 3 ID : 3398938387

Option 4 ID : 3398938388

Status : Not Answered

Chosen Option : --

Q.35 Is Jacobi's necessary condition satisfied by the admissible extremal for

$$J[y] = \int_0^b (y'^2 - y^2) dx?$$

1. Yes, for all values of b
2. No, for any value of b
3. Yes, for $b < \pi$
4. Yes, for $b \geq \pi$

Options

1. 1
2. 2
3. 3
4. 4

Question Type : MSQ

Question ID : 3398932155

Option 1 ID : 3398938389

Option 2 ID : 3398938390

Option 3 ID : 3398938391

Option 4 ID : 3398938392

Status : Not Answered

Chosen Option : --

Q.36

Let $p \in C^1[0,1]$, $q \in C[0,1]$ and p be positive. Consider a solution $x > 0$ of the boundary value problem

$p(t)x''(t) + p'(t)x'(t) + q(t)x(t) = \beta x(t)$, $t \in (0,1)$, $x'(0) = 0 = x'(1)$, where $\beta \in \mathbb{R}$. If $\int_0^1 q(t)dt > 0$ then which of the following statements are true?

1. $\beta \in (-4, -1]$
2. $\beta \in (-1, 0]$
3. $\beta \in (0, \infty)$
4. $\beta \in (-\infty, -4]$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932150

Option 1 ID : 3398938369

Option 2 ID : 3398938370

Option 3 ID : 3398938371

Option 4 ID : 3398938372

Status : Not Answered

Chosen Option : --

Q.37 Let $K: [a, b] \times [a, b] \rightarrow \mathbb{R}$ be a nonzero continuous map with $K(x, t) = K(t, x)$, for $t, x \in [a, b]$ and $I: C[a, b] \rightarrow C[a, b]$ be the identity operator. Define $T, S: C[a, b] \rightarrow C[a, b]$ such that for $\varphi \in C[a, b]$, $x \in [a, b]$

$$(T\varphi)(x) = \int_a^x K(x, t)\varphi(t)dt, (S\varphi)(x) = \int_a^b K(x, t)\varphi(t)dt.$$

Then which of the following statements are true?

1. $\forall \lambda \in \mathbb{R} \setminus \{0\}$, the only solution of $(I - \lambda T)\varphi = 0$ is $\varphi \equiv 0$
2. $\forall \lambda \in \mathbb{R} \setminus \{0\}$, $\forall f \in C[a, b]$, there exists a unique solution of $(I - \lambda T)\varphi = f$
3. $\forall \lambda \in \mathbb{R} \setminus \{0\}$, $\forall f \in C[a, b]$, there exists a unique solution of $(I - \lambda S)\varphi = f$
4. $\exists \lambda \in \mathbb{R} \setminus \{0\}$ such that $(I - \lambda S)\varphi = 0$ has a nonzero solution

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932157

Option 1 ID : 3398938397

Option 2 ID : 3398938398

Option 3 ID : 3398938399

Option 4 ID : 3398938400

Status : Not Answered

Chosen Option : --

Q.38

Let $B(0,1) = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$. Consider $u \in C^1(\overline{B(0,1)})$, satisfying

$\Delta u + \lambda u = 0$ in $B(0,1)$, $\frac{\partial u}{\partial n} = 0$ on $\partial(B(0,1))$. Then

1. $\lambda \geq 0$
2. $\lambda < 0$
3. $\lambda = 0$ implies $u = 0$
4. $\lambda = 0$ implies u is a constant

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932152

Option 1 ID : 3398938377

Option 2 ID : 3398938378

Option 3 ID : 3398938379

Option 4 ID : 3398938380

Status : Not Answered

Chosen Option : --

Q.39 Consider the integration formula

$$\int_{x_0}^{x_1} f(x) dx = \frac{h}{2} [f(x_0) + f(x_1)] + ph^2 (f'(x_0) - f'(x_1)),$$

where $h = x_1 - x_0$. Then the constant p such that the above formula gives the exact value for the highest degree polynomial and the degree d of the corresponding polynomial are given by

1. $p = \frac{1}{6}, d = 4$
2. $p = \frac{1}{12}, d = 3$
3. $p = \frac{1}{6}, d = 3$
4. $p = \frac{1}{12}, d = 4$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932153

Option 1 ID : 3398938381

Option 2 ID : 3398938382

Option 3 ID : 3398938383

Option 4 ID : 3398938384

Status : Not Answered

Chosen Option : --

Q.40

Let $g: \mathbb{R} \rightarrow \mathbb{R}$ be locally Lipschitz and $xy(x) \leq 0, \forall x \in \mathbb{R}$. Then which of the following statements are true for the initial value problem

$$x'(t) = g(x(t)), \quad x(0) = x_0 \in \mathbb{R}.$$

1. There exists a local solution
2. For any $x_0 \in \mathbb{R}$, the solution exists on $[0, \infty)$
3. There is a finite time blow-up for some x_0
4. All the solutions are bounded on $[0, \infty)$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932148

Option 1 ID : 3398938361

Option 2 ID : 3398938362

Option 3 ID : 3398938363

Option 4 ID : 3398938364

Status : Not Answered

Chosen Option : --

Q.41 Let $f: \mathbb{R}^2 \rightarrow (-\infty, 0]$ be such that $f(0,0) = 0$. Consider the following system of ODEs

$$x'(t) = f(x(t), y(t)), \quad y'(t) = y^2(t), t > 0.$$

A set $S \subseteq \mathbb{R}^2$ is said to be positively invariant set if the following holds.

Assume that $(x(0), y(0)) \in S$. Then for $t \geq 0$, if the solution $(x(t), y(t))$ exists, then $(x(t), y(t)) \in S$. Then which of the following sets are positively invariant

1. $\{(x, y) \in \mathbb{R}^2: x \leq 0, y \leq 0\}$
2. $\{(x, y) \in \mathbb{R}^2: y \geq 0\}$
3. $\{(x, y) \in \mathbb{R}^2: x \leq 0, y \leq -1\}$
4. $\{(x, y) \in \mathbb{R}^2: x \leq 0, y \geq 0\}$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932149

Option 1 ID : 3398938365

Option 2 ID : 3398938366

Option 3 ID : 3398938367

Option 4 ID : 3398938368

Status : Not Answered

Chosen Option : --

Q.42

If $u(x, y)$ is the solution of the following problem

$$u \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 1, \text{ such that } u(x_0(s), y_0(s)) = u_0(s), \text{ where}$$

$$x_0(s) = s, y_0(s) = s, u_0(s) = s/2. \text{ Then } u(2,1) \text{ is equal to}$$

1. $1/2$
2. $-(1/2)$
3. 1
4. -1

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932151

Option 1 ID : 3398938373

Option 2 ID : 3398938374

Option 3 ID : 3398938375

Option 4 ID : 3398938376

Status : Not Answered

Chosen Option : --

Q.43 Consider two samples of sizes n_1 and n_2 drawn from a finite population of size $N \geq 10$, using SRSWR and SRSWOR respectively. Suppose population coefficients of variation of the two sample means are the same. Assume that the population mean is positive. Which of the following are true?

1. $n_1 \geq n_2$
2. If $n_1 = n_2$, then $n_2 = 1$
3. $N(n_1 - n_2) = n_2(n_1 - 1)$
4. $n_2(N - 1) = n_1(N - n_2)$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932173

Option 1 ID : 3398938461

Option 2 ID : 3398938462

Option 3 ID : 3398938463

Option 4 ID : 3398938464

Status : Not Answered

Chosen Option : --

Q.44

Let $X_1, X_2, \dots, X_m$ be a random sample from a continuous distribution F and let $Y_1, \dots, Y_n$ be a random sample from a continuous distribution G . We want to test the hypothesis $H_0: F(x) = G(x) \forall x$ versus $H_1: G(x) = F(x - \theta), \forall x$, for some fixed $\theta (\neq 0)$. Assume that $X_1, \dots, X_m, Y_1, \dots, Y_n$ are independent. Let R_{m+j} denote the rank of Y_j in the combined ordering and $S = \sum_{j=1}^n R_{m+j}$. Which of the following are true?

1. Under $H_0, P[R_{m+j} = k] = \frac{1}{n+m}$ for $k = 1, 2, \dots, n+m$
2. If the observed value of S is $\frac{n(n+m+1)}{2}$, then the large sample test does not reject H_0 at level α , for all $0 < \alpha < 1$
3. If $\theta > 0$, a large value of S supports H_1
4. The assumption $n = m$ is needed to obtain the asymptotic null distribution of $\frac{S - E_{H_0}[S]}{\sqrt{\text{Var}_{H_0}(S)}}$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932169

Option 1 ID : 3398938445

Option 2 ID : 3398938446

Option 3 ID : 3398938447

Option 4 ID : 3398938448

Status : Not Answered

Chosen Option : --

Q.45 Consider a 2-way analysis of variance experiment with 4 rows, 3 columns and a single observation per cell. The experiment was carried out keeping cells (1,1), (2,2), (3,2) and (4,1) empty. Let $\alpha_i, i = 1, 2, 3, 4$ be the row effects and $\beta_j, j = 1, 2, 3$ be the column effects. Which of the following statements are true?

1. Only 2 linearly independent elementary contrasts of α are estimable
2. the elementary contrast $\beta_1 - \beta_2$ is estimable
3. the contrast $\alpha_1 - \alpha_2 + \alpha_3 - \alpha_4$ is not estimable
4. the contrast $\alpha_1 - \alpha_3$ is estimable

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932171

Option 1 ID : 3398938453

Option 2 ID : 3398938454

Option 3 ID : 3398938455

Option 4 ID : 3398938456

Status : Not Answered

Chosen Option : --

Q.46

X is a random variable with normal distribution having mean 1 and variance 4.

Which of the following are true?

1. $E[X^2] = 18$
2. $\text{Var}(X^2) = 48$
3. $E[\min(X, 3)] < 1$
4. $E[(X + 1)^2] \leq E[(X - 1)^2]$

Options

1. 1
2. 2
3. 3
4. 4

Question Type : MSQ

Question ID : 3398932164

Option 1 ID : 3398938425

Option 2 ID : 3398938426

Option 3 ID : 3398938427

Option 4 ID : 3398938428

Status : Not Answered

Chosen Option : --

Q.47 P is the transition matrix of a finite state space Markov chain. Which of the following statements are necessarily true? (Below I denotes the identity matrix of the same size as P).

1. If P is irreducible then P^2 is irreducible
2. If P is not irreducible then P^2 is not irreducible
3. If P is irreducible then $\frac{I+P}{2}$ is irreducible
4. If P is irreducible and aperiodic, then P^3 is irreducible

Options

1. 1
2. 2
3. 3
4. 4

Question Type : MSQ

Question ID : 3398932162

Option 1 ID : 3398938417

Option 2 ID : 3398938418

Option 3 ID : 3398938419

Option 4 ID : 3398938420

Status : Not Answered

Chosen Option : --

Q.48

Consider a LP problem:

Maximize $z = \underline{c}'\underline{x}$ subject to $A\underline{x} \leq \underline{b}$, $\underline{x} \geq 0$.

Which of the following are true?

1. If the primal has an optimal solution, then the dual also has an optimal solution.
2. The primal can have an optimal solution and the dual may have no feasible solution.
3. The dual can have an optimal solution and the primal may have no feasible solution.
4. If the primal has no feasible solution then the dual also has no feasible solution.

Options

1. 1
2. 2
3. 3
4. 4

Question Type : MSQ

Question ID : 3398932176

Option 1 ID : 3398938473

Option 2 ID : 3398938474

Option 3 ID : 3398938475

Option 4 ID : 3398938476

Status : Not Answered

Chosen Option : --

Q.49 Let X and Y be two random variables on a probability space. Consider the following statements

- A. Each of X and Y is normally distributed
- B. X and Y are independent
- C. The conditional distribution of X given $Y = y$ and the conditional distribution of Y given $X = x$ are normal for all x and y .

Then for (X, Y) to have a bivariate normal distribution

1. A alone is sufficient
2. A and B together are sufficient
3. A and B are necessary
4. C is necessary

Options

1. 1
2. 2
3. 3
4. 4

Question Type : MSQ

Question ID : 3398932172

Option 1 ID : 3398938457

Option 2 ID : 3398938458

Option 3 ID : 3398938459

Option 4 ID : 3398938460

Status : Not Answered

Chosen Option : --

Suppose Y_1 and Y_2 are i.i.d. Uniform(0, θ), where $\theta > 0$. Define $M = \max\{Y_1, Y_2\}$. Which of the following are confidence intervals for θ with confidence coefficient 0.91?

1. $\left[M, \frac{10}{3}M\right]$
2. $\left[\frac{9}{10}M, \frac{10}{3}M\right]$
3. $\left[\frac{1}{2}M, \frac{10}{3}M\right]$
4. $\left[\frac{1}{2}M, 2M\right]$

Options 1. 1

2. 2
3. 3
4. 4

Question Type : MSQ

Question ID : 3398932168

Option 1 ID : 3398938441

Option 2 ID : 3398938442

Option 3 ID : 3398938443

Option 4 ID : 3398938444

Status : Not Answered

Chosen Option : --

Q.51 Let $X_1, X_2, \dots, X_n$ be i.i.d. random variables with probability mass function given by $p_\theta(x) = \left(\frac{\theta}{2}\right)^{|x|} (1-\theta)^{1-|x|}$ for $x = -1, 0, 1$, where $\theta \in (0, 1)$. Then

1. $\sum_{i=1}^n |X_i|$ is complete sufficient for θ
2. $\frac{1}{n} \sum_{i=1}^n X_i$ is MLE of θ
3. $\frac{1}{n} \sum_{i=1}^n |X_i|$ is MLE of θ
4. $\frac{1}{n} \sum_{i=1}^n X_i$ is unbiased for θ

Options 1. 1

2. 2
3. 3
4. 4

Question Type : MSQ

Question ID : 3398932165

Option 1 ID : 3398938429

Option 2 ID : 3398938430

Option 3 ID : 3398938431

Option 4 ID : 3398938432

Status : Not Answered

Chosen Option : --

For a 2^4 factorial experiment with 4 treatments A, B, C, D each at 2 levels, 4 blocks of size 4 each were available. If the key block is

(1) (bc) (acd) (abd)

which of the following treatment combinations are confounded?

1. ABC
2. ABD
3. ACD
4. BCD

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSO

Question ID : 3398932174

Option 1 ID : 3398938465

Option 2 ID : 3398938466

Option 3 ID : 3398938467

Option 4 ID : 3398938468

Status : Not Answered

Chosen Option : --

Q.53 Let N_t be the number of customers that enter my shop up-to time t . Assume that $(N_t)_{t \geq 0}$ is Poisson process with parameter $\lambda = 5$. Whenever a customer enters, a fair die (usual die with six faces) is rolled. Let X be the number of 1's up-to time 10 and Y be the number of 2's up-to time 10. Which of the following are correct?

1. X, Y are independent
2. $X + Y \leq 50$
3. The joint distribution of (X, Y) is multinomial
4. X, Y are Poisson random variables

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSO

Question ID : 3398932177

Option 1 ID : 3398938477

Option 2 ID : 3398938478

Option 3 ID : 3398938479

Option 4 ID : 3398938480

Status : Not Answered

Chosen Option : --

Q.54

Let $X_1, X_2, \dots, X_n$ be i.i.d. from Poisson (λ) where $\lambda > 0$. Consider testing

$H_0: \lambda \leq \lambda_0$ versus $H_1: \lambda > \lambda_0$.

Let Test A be the test that rejects H_0 if $\sum_{i=1}^n X_i > k$ where k is such that $P(\sum_{i=1}^n X_i > k | \lambda = \lambda_0) = \alpha$.

Let Test B be the test that rejects H_0 if $\sum_{i=1}^n X_i < k$ where k is such that $P(\sum_{i=1}^n X_i < k | \lambda = \lambda_0) = \alpha$.

Then,

1. Test A is the likelihood ratio test of size α
2. Test B is the likelihood ratio test of size α
3. Test A is a UMP test of size α
4. Test B is a UMP test of size α

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ

Question ID : 3398932167

Option 1 ID : 3398938437

Option 2 ID : 3398938438

Option 3 ID : 3398938439

Option 4 ID : 3398938440

Status : Not Answered

Chosen Option : --

Q.55 Suppose the hazard rate function of the lifetime T of a system is $h(t) = t^2$ for $t > 0$. Then which of the following are true?

1. The probability that the system fails prior to 3 time units is $\frac{1}{9}$
2. The probability that the system fails at time $\frac{1}{3}$ given $T \geq \frac{1}{3}$ is $\frac{1}{9}$
3. The probability density function of T is $f(x) = t^2 e^{-(t^3/3)}$ for $t > 0$
4. $E[T^3] = 3$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : MSQ

Question ID : 3398932175

Option 1 ID : 3398938469

Option 2 ID : 3398938470

Option 3 ID : 3398938471

Option 4 ID : 3398938472

Status : Not Answered

Chosen Option : --

Q.56

Then which of the following are true?

1. $Z_n \rightarrow \mu^2$ with probability one
2. If $\mu \neq 0$ then $Z_n \rightarrow \infty$ with probability one
3. If $\mu = 0$ then $\frac{Z_n}{\sigma^2}$ converges in distribution
4. If $\mu = 0$ then $\frac{Z_n}{\sigma^2}$ converges in distribution to standard normal

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSO

Question ID : 3398932161

Option 1 ID : 3398938413

Option 2 ID : 3398938414

Option 3 ID : 3398938415

Option 4 ID : 3398938416

Status : Not Answered

Chosen Option : --

Q.57 Let $X_1, X_2, \dots, X_n$ be a random sample from an exponential (λ) distribution, $\lambda > 0$, i.e.,

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Let $\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$. Then which of the following are correct?

1. $\bar{X}_n$ is MLE of λ
2. $\frac{1}{\bar{X}_n}$ is MLE of λ
3. $\frac{1}{n} \left(\frac{1}{X_1} + \frac{1}{X_2} + \dots + \frac{1}{X_n} \right)$ is MLE of λ
4. $\frac{1}{\bar{X}_n}$ is an unbiased estimator of λ

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSO

Question ID : 3398932166

Option 1 ID : 3398938433

Option 2 ID : 3398938434

Option 3 ID : 3398938435

Option 4 ID : 3398938436

Status : Not Answered

Chosen Option : --

Q.58

Which of the following conditions imply that $X_n \rightarrow X$ in distribution as $n \rightarrow \infty$?

1. $X_n^2 \rightarrow X^2$ with probability one
2. $E[|X_n - X|^2] \rightarrow 0$
3. $E[e^{i\lambda X_n}] \rightarrow E[e^{i\lambda X}]$ for all $\lambda \in \mathbb{R}$
4. $X_n \rightarrow X$ with probability one

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932160

Option 1 ID : 3398938409

Option 2 ID : 3398938410

Option 3 ID : 3398938411

Option 4 ID : 3398938412

Status : Not Answered

Chosen Option : --

Q.59 Let $X_1, X_2, \dots$ be i.i.d. random variables from a distribution F . Let

$$Y_n = \min\{X_1, \dots, X_n\}.$$

If nY_n converges in distribution to exponential distribution with mean $1/2$, then F could be

1. $N(0, 1)$
2. $\text{Uniform}(0, 1)$
3. $\text{Uniform}(0, \frac{1}{2})$
4. $\text{Uniform}(0, 2)$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : MSQ

Question ID : 3398932163

Option 1 ID : 3398938421

Option 2 ID : 3398938422

Option 3 ID : 3398938423

Option 4 ID : 3398938424

Status : Not Answered

Chosen Option : --

Q.60 Let $(Y_i, X_i), i = 1, \dots, n$ be observations where X_i are the regressors and Y_i are response variables which follow Bernoulli distribution with probability of success $\pi(X_i)$, $i = 1, \dots, n$. Suppose the log-odds ratio is a linear function with intercept α and slope β . Which of the following statements are true?

1. If $\beta > 0$ then $\pi(x)$ is an increasing function of x
2. $E[Y | X] = e^{\alpha + \beta X}$
3. If $x = -\frac{\alpha}{\beta}$ with $\beta \neq 0$, then $\pi(x) = \frac{1}{2}$
4. If $x = -\frac{\alpha}{\beta}$ then $\pi(x + c) = \pi(x - c)$ for any c .

Options 1. 1

2. 2

3. 3

4. 4

Question Type : **MSQ**

Question ID : **3398932170**

Option 1 ID : **3398938449**

Option 2 ID : **3398938450**

Option 3 ID : **3398938451**

Option 4 ID : **3398938452**

Status : **Not Answered**

Chosen Option : --



SNO	QUESTION ID	CORRECT OPTION(S)	SNO	QUESTION ID	CORRECT OPTION(S)	SNO	QUESTION ID	CORRECT OPTION(S)	SNO	QUESTION ID	CORRECT OPTION(S)
001	3398932058	3398938003	021	3398932078	3398938083	061	3398932118	3398938244			
002	3398932059	3398938007	022	3398932079	3398938085	062	3398932119	3398938245 , 3398938246 , 3398938248			
003	3398932060	3398938009	023	3398932080	3398938091	063	3398932120	3398938249			
004	3398932061	3398938013	024	3398932081	3398938096	064	3398932121	3398938254 , 3398938255			
005	3398932062	3398938019	025	3398932082	3398938100	065	3398932122	3398938258 , 3398938260			
006	3398932063	3398938021	026	3398932083	3398938102	066	3398932123	3398938261 , 3398938262 , 3398938263			
007	3398932064	3398938025	027	3398932084	3398938106	067	3398932124	3398938265 , 3398938268			
008	3398932065	3398938030	028	3398932085	3398938111	068	3398932125	3398938269 , 3398938270 , 3398938272			
009	3398932066	3398938035	029	3398932086	3398938115	069	3398932126	3398938275			
010	3398932067	3398938038	030	3398932087	3398938118	070	3398932127	3398938277			
011	3398932068	3398938041	031	3398932088	3398938121	071	3398932128	3398938282 , 3398938283 , 3398938284			
012	3398932069	3398938048	032	3398932089	3398938125	072	3398932129	3398938285 , 3398938286			
013	3398932070	3398938050	033	3398932090	3398938132	073	3398932130	3398938289 , 3398938290			
014	3398932071	3398938055	034	3398932091	3398938136	074	3398932131	3398938293			
015	3398932072	3398938058	035	3398932092	3398938137	075	3398932132	3398938298 , 3398938299			
016	3398932073	3398938062	036	3398932093	3398938143	076	3398932133	3398938304			
017	3398932074	3398938066	037	3398932094	3398938146	077	3398932134	3398938307 , 3398938308			
018	3398932075	3398938070	038	3398932095	3398938151	078	3398932135	3398938309 , 3398938310 , 3398938311			
019	3398932076	3398938073	039	3398932096	3398938155	079	3398932136	3398938315			
020	3398932077	3398938079	040	3398932097	3398938160	080	3398932137	3398938317			
			041	3398932098	3398938161	081	3398932138	3398938323			
			042	3398932099	3398938167	082	3398932139	3398938328			
			043	3398932100	3398938171	083	3398932140	3398938330 , 3398938332			
			044	3398932101	3398938174	084	3398932141	3398938333 , 3398938334			
			045	3398932102	3398938179	085	3398932142	3398938340			
			046	3398932103	3398938181	086	3398932143	3398938342			
			047	3398932104	3398938187	087	3398932144	3398938346 , 3398938347			
			048	3398932105	3398938189	088	3398932145	3398938349 , 3398938351 , 3398938352			
			049	3398932106	3398938195	089	3398932146	3398938355 , 3398938356			
			050	3398932107	3398938200	090	3398932147	3398938357 , 3398938359			
			051	3398932108	3398938204	091	3398932148	3398938361 , 3398938362 , 3398938364			
			052	3398932109	3398938205	092	3398932149	3398938365 , 3398938366 , 3398938368			
			053	3398932110	3398938211	093	3398932150	3398938371			
			054	3398932111	3398938215	094	3398932151	3398938374			
			055	3398932112	3398938219	095	3398932152	3398938377 , 3398938380			
			056	3398932113	3398938224	096	3398932153	3398938382			

057	3398932114	3398938227	097	3398932154	3398938385 , 3398938387
058	3398932115	3398938231	098	3398932155	3398938391
059	3398932116	3398938235	099	3398932156	3398938393 , 3398938396
060	3398932117	3398938238	100	3398932157	3398938397 , 3398938398 , 3398938400
			101	3398932158	3398938401 , 3398938402 , 3398938403
			102	3398932159	3398938406 , 3398938407
			103	3398932160	3398938410 , 3398938411 , 3398938412
			104	3398932161	3398938414 , 3398938415
			105	3398932162	3398938418 , 3398938419 , 3398938420
			106	3398932163	3398938423
			107	3398932164	3398938426 , 3398938427
			108	3398932165	3398938429 , 3398938431
			109	3398932166	3398938434
			110	3398932167	3398938437 , 3398938439
			111	3398932168	3398938441 , 3398938442 , 3398938443
			112	3398932169	3398938445 , 3398938446 , 3398938447
			113	3398932170	3398938449 , 3398938451
			114	3398932171	3398938454 , 3398938456
			115	3398932172	3398938458 , 3398938460
			116	3398932173	3398938461 , 3398938462 , 3398938463 , 3398938464
			117	3398932174	3398938465 , 3398938468
			118	3398932175	3398938471 , 3398938472
			119	3398932176	3398938473
			120	3398932177	3398938477 , 3398938480