Ordinary Differential Equations:

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1. Consider the initial value problem (IVP)

$$y'(t) = e^{-t^2} y(t), \quad y(e) = 0.5,$$

. Then, the IVP

   (a) has infinitely many solutions
   (b) has a solution which is not defined at some $t \in \mathbb{R}$
   (c) has a unique solution in $\mathbb{R}$
   (d) has no solution

2. Let $y_1(x)$ and $y_2(x)$ be the solutions of the differential equation $\frac{dy}{dx} = y + 17$ with initial conditions $y_1(0) = 0, y_2(0) = 1$. Then

   (a) y_1 and y_2 will never intersect.
   (b) y_1 and y_2 will intersect $x = 17$.
   (c) y_1 and y_2 will intersect $x = e$
   (d) y_1 and y_2 will intersect $x = 1$

3. Consider the differential equation

$$\frac{dy}{dx} = y^2, \qquad (x, y) \in \mathbb{R} \times \mathbb{R}.$$

Then

   (a) all solutions of the differential equations are defined on $(-\infty, \infty)$.
   (b) no solutions of the differential equations are defined on $(-\infty, \infty)$.
   (c) the solution of the differential equations satisfying the initial condition $y(x_0) = y_0, \quad y_0 > 0$, is defined on $(-\infty, x_0 + \frac{1}{y_0})$.
   (d) the solution of the differential equations satisfying the initial condition $y(x_0) = y_0, \quad y_0 > 0$, is defined on $(x_0 - \frac{1}{y_0}, \infty)$.

4. The initial value problem $\dot{x}(t) = 3x^{2/3}, \quad x(0) = 0$, in an interval around $t = 0$, has

   (a) no solution
   (b) a unique solution
   (c) finitely many linearly independent solutions
   (d) infinitely many linearly independent solutions

5. Consider the equation

$$\frac{dy}{dt} = (1 + f^2(t))y(t), \quad y(0) = 1. \quad t \geq 0,$$

where f is bounded continuous functions on $[0, \infty)$. Then

   (a) This equation admits a unique solution $y(t)$ and further $\lim_{t\to\infty} y(t)$ exists and is finite.
   (b) This equation admits two linearly independent solutions.
   (c) This equation admits a bounded solution for which $\lim_{t\to\infty} y(t)$ does not exist.
   (d) This equation admits a unique solution $y(t)$ and further, $\lim_{t\to\infty} y(t) = \infty$.

6. Let $y : \mathbb{R} \to \mathbb{R}$ satisfy the initial value problem

$$y'(t) = 1 - y^2(t), \quad y(0) = 0, \quad t \in \mathbb{R}.$$

Then

   (a) $y(t_1) = 1$ for some $t_1 \in \mathbb{R}$.
   (b) $y(t) > -1$ for all $t \in \mathbb{R}$.
   (c) y is strictly increasing in $\mathbb{R}$.
   (d) y is increasing in $(0, 1)$ and decreasing in $(1, \infty)$.

7. The initial value problem $y' = 2\sqrt{y}, \quad y(0) = a$, has

   (a) a unique solution if $a < 0$

   (b) no solution if $a > 0$

   (c) infinitely many solutions if $a = 0$

   (d) a unique solution if $a \geq 0$

8. Consider the ODE on $y'(x) = f(y(x))$. If f is an even function and y is an odd function, then

   (a) $-y(-x)$ is also a solution.

   (b) $y(-x)$ is also a solution.

   (c) $-y(x)$ is also a solution.

   (d) $y(x)y(-x)$ is also a solution.

9. Consider the solution of the ordinary differential equation

$$y'(t) = -y^3 + y^2 + 2y, \quad \text{subject to} \quad y(0) = y_0 \in (0, 2).$$

   Then $\lim_{t\to\infty} y(t)$ belongs to

   (a) $\{-1, 0\}$

   (b) $\{-1, 2\}$

   (c) $\{0, 2\}$

   (d) $\{0, \infty\}$

10. Consider the ordinary differential equation

$$y' = y(y-1)(y-2).$$

   Which of the following statements is true?

   (a) If $y(0) = 0.5$, then y is decreasing

   (b) If $y(0) = 1.2$, then y is increasing

   (c) If $y(0) = 2.5$, then y is unbounded

   (d) If $y(0) < 0$, then y is bounded below

11. Assume that $a : [0, \infty) \to \mathbb{R}$ is a continous function. Consider the ordinary differential equation

$$y'(x) = a(x)y(x), \quad x > 0, \quad y(0) = y_0 \neq 0.$$

   which of the following statements are true?

   (a) If $\int_0^\infty |a(x)|dx < \infty$, then y is bounded

   (b) If $\int_0^\infty |a(x)|dx < \infty$, then $\lim_{x\to\infty} y(x)$ exists

   (c) If $\lim_{x\to\infty} a(x) = 1$, then $\lim_{x\to\infty} |y(x)| = \infty$

   (d) If $\lim_{x\to\infty} a(x) = 1$, then y is monotone